Robust Generalized Predictive Control with Uncertainty Quantification

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This paper presents a robust predictive control method with uncertainty quantification for problems, such as online aircraft flutter suppression. The proposed approach begins by identifying the multistep output prediction matrices under various test conditions from online time-domain input/output data. Singular value decomposition is then used to characterize and quantify the parameter uncertainties of the prediction matrices. The predicted response error due to parameter uncertainties is integrated into the predictive control design process for robustness. The proposed method provides an innovative and efficient way to incorporate the quantified uncertainty in predictive control design with a linear process. The proposed approach is demonstrated by application to the Benchmark Active Controls Technology wind-tunnel model.

I. Introduction

PREDICTIVE control refers to a strategy wherein the decision for the current control action is based on minimization of a quadratic objective function that involves a prediction of the system state at some number of time steps in the future. Model-based predictive control approaches have attracted much attention, both in academia and industry, due to their relatively simple time-domain formulation and good performance [1]. Among these methods, generalized predictive control (GPC) [2] has received widespread acceptance. There are two fundamental steps involved in GPC implementation: 1) identification of the system and 2) use of the identified model to design a controller. GPC has been successfully applied for actively controlling the swashplate of tiltrotor aircraft to enhance aeroelastic stability in both helicopter and airplane modes of flight at NASA Langley Research Center (LaRC) [3], where the controller is updated with a linear process, based on the identified prediction matrices and measured input/output data. Studies demonstrated that GPC provides an effective tool for adaptive modeling and control of gust load alleviation [3–5].

The aeroelastic flutter of a flight vehicle is a dynamic instability associated with the interaction of aerodynamic, elastic, and inertial forces [6]. Flutter can lead to a catastrophic structural failure of the flight vehicle, and the flutter problem is generally accepted as a problem of primary concern in the design of aircraft structures. Active control of aeroelastic phenomena at transonic speeds is a crucial technology for future aircraft design [7]. Recently, flutter suppression techniques have been investigated with the Benchmark Active Controls Technology (BACT) wind-tunnel model [8–11], which was developed by the researchers at LaRC to address the flutter problem.

A fundamental issue of predictive control is its robustness to model uncertainty. Current GPC techniques do not incorporate the quantified model uncertainty in the control design process, where the quadratic objective function does not include the error term related to model uncertainty. The integration of the quantified model uncertainty into GPC is one major issue addressed in this paper. The main objective of this paper is to develop a robust predictive control design process capable of quantifying model uncertainties, which can be applied to online aircraft flutter suppression under changing flight conditions. The proposed robust generalized

Nomenclature

\[ A, B, T \] = multistep output prediction matrices  
\[ A_0, B_0, T_0 \] = nominal prediction matrices  
\[ A_i, B_i, T_i \] = uncertainty prediction matrices  
\[ d_i, d_u \] = known and unknown disturbances  
\[ J \] = cost function for controller design  
\[ Q \] = weighting matrix for prediction error  
\[ Q_u \] = weighting matrix for uncertainty error  
\[ R \] = weighting matrix for control force  
\[ S \] = singular value matrix  
\[ s_i \] = singular values  
\[ U \] = matrix of principal directions  
\[ U, \tilde{V} \] = left and right matrices of singular value decomposition  
\[ u, y \] = input and output vectors  
\[ u_i \] = control input vector  
\[ V, Y \] = data matrices of input and output  
\[ W \] = normalized weighting matrix for parameter variations  
\[ w_1 \] = diagonal elements of \( R \)  
\[ w_2 \] = diagonal elements of \( Q_u \)  
\[ Y \] = matrix of observer Markov parameters  
\[ \alpha_i, \beta_i \] = observer Markov parameters  
\[ \alpha^r, \beta^r \] = control gain matrices  
\[ \alpha^c, \beta^c \] = designed control gain matrices  
\[ \Gamma \] = coordinate matrix  
\[ \gamma_i \] = uncertainty coordinates  
\[ \Delta \varepsilon_i \] = prediction error from uncertainty coordinate  
\[ \Delta \theta \] = matrix of parameter variations  
\[ \Delta \theta_i \] = deviation of identified parameter vector  
\[ \varepsilon_0 \] = prediction error of nominal system  
\[ \theta \] = parameter vector  
\[ \theta_i \] = identified parameter vector  
\[ \psi_i \] = principal direction

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predictive control (RGPC) method has three fundamental steps in the design process. The first step is to perform online system identification to compute output prediction matrices to map input-output data under different test conditions. The second step is to use singular value decomposition [12,13] (SVD) to characterize and quantify parameter uncertainties of the identified output prediction matrices, which are identified directly from online input/output data [5] under various flight conditions. Singular values and vectors are used to compute uncertainty coordinates and their corresponding principal directions. The standard deviation of each uncertainty coordinate is computed to determine the distribution and number of dominant uncertainty coordinates. The third step is to integrate the output response error due to the identified dominant uncertainty coordinates into the cost function for predictive control design [5]. Robustness design is achieved by tuning the weighting of the error term corresponding to the quantified uncertainty. The proposed method provides an innovative and efficient way to incorporate the quantified uncertainty in predictive control design with a linear process that requires little computational effort. The proposed approach is demonstrated by application to the BACT wind-tunnel model with flight condition variations.

II. Basic Formulation

The essential features of the adaptive control process used in the present GPC investigation are depicted in Fig. 1. The system (plant) has r control inputs \( u \) and m measured outputs \( y \) and is subject to unknown external disturbances \( d \). Measurement noise is also present. There are two fundamental steps involved: 1) identification of the system and 2) use of the identified model to design a controller. A finite-difference model in the form of an auto-regressive moving average with exogenous input (ARX) model is used here. This model is used for both system identification and controller design. System identification is done online in the presence of any disturbances acting on the system, as indicated in the center box of the diagram in Fig. 1. An estimate of the disturbance model is embedded in the identified system model and does not have to be modeled separately. This approach represents a case of feedback with embedded feedforward. Because the disturbance information is embedded in the feedforward control parameters, there is no need for measurement of the disturbance signal. The parameters of the identified model are used to compute the predictive control law. For identification, a random excitation (sometimes called dither) is applied initially with the control force \( u \) equal to zero to identify the open-loop system. Dither is added to the closed-loop control input \( u \), if it is necessary to reidentify the system while operating in the closed-loop mode.

The relationship between the input and output time histories of a linear system is described by the time-domain ARX finite-difference model as follows

\[
y(k) + \alpha_1 y(k-1) + \alpha_2 y(k-2) + \cdots + \alpha_p y(k-p) = \beta_0 u(k) + \beta_1 u(k-1) + \beta_2 u(k-2) + \cdots + \beta_p u(k-p)
\]  

(1)

This equation states that the current output \( y(k) \) at time step \( k \) can be estimated by using \( p \) sets of the previous output and input measurements, \( y(k-1), \ldots, y(k-p) \) and \( u(k-1), \ldots, u(k-p) \), and the current input measurement \( u(k) \). The integer \( p \) is called the order of the ARX model. The coefficient matrices \( \alpha_i \) and \( \beta_j \) appearing in this equation are referred to as observer Markov parameters (OMP) or ARX parameters and are the quantities to be determined by the identification algorithm. Closed-loop robustness is enhanced by performing the system identification in the presence of the external disturbances acting on the system, thereby ensuring that disturbance information will be incorporated into the system model. The goal of system identification is to determine the OMP based on input and output data. The OMP may be determined by any identification techniques that return an ARX model of the system.

The ARX model is used to design the controller and leads to a control law that, in the case of a regulator problem, has the general form given as follows

\[
u_i(k) = \alpha_0^T y(k-1) + \alpha_1^T y(k-2) + \cdots + \alpha_p^T y(k-p) + \beta_0^T u_i(k-1) + \beta_1^T u_i(k-2) + \cdots + \beta_p^T u_i(k-p)
\]

(2)

This equation indicates that the current control input \( u_i(k) \) can be computed using \( p \) sets of the previous input and output measurements and the control gain matrices \( \alpha_i \) and \( \beta_i \). The derivation of the parameters appearing in the system identification and control law equations is described in the following paragraphs.

System identification in the presence of the operational disturbances acting on the system is the first of the two major computational steps. The external disturbances acting on the system are assumed to be unknown (unmeasurable). The order of the ARX model \( p \) and the number of time steps must be specified. Some guidelines for their selection are given in this section.

To ensure a sufficiently rich input signal for identification, the system is excited with band-limited white noise at all \( r \) control inputs simultaneously while measuring the responses at \( m \) locations. The digitized input and output time histories \( u \) and \( y \) are then used to form the data matrices \( Y \), \( V \) as follows

\[
Y = [ y(0) \quad y(1) \quad y(2) \quad \cdots \quad y(p) \quad \cdots \quad y(\ell - 1) ]
\]

(3)

\[
V = 
\begin{bmatrix}
0 \quad u(1) \quad u(2) \quad \cdots \quad u(p) \quad \cdots \quad u(\ell - 1) \\
v(0) \quad v(1) \quad v(p - 1) \quad \cdots \quad v(\ell - 2) \\
v(0) \quad \cdots \quad v(p - 2) \quad \cdots \quad v(\ell - 3) \\
\vdots \quad \vdots \quad \cdots \quad \vdots \\
v(0) \quad \cdots \quad v(\ell - p - 1)
\end{bmatrix}
\]

(4)

where the vector \( v(k) \) of dimension \( r + m \times 1 \) is defined as

\[
v(k) = \begin{bmatrix} u(k) \\ y(k) \end{bmatrix}
\]

(5)

Note that \( y \) is a \( m \times \ell \) matrix, and \( V \) is a \( [r + (r + m)p] \times \ell \) matrix. Writing the discrete-time finite-difference Eq. (1) as a sequence of time steps \( k = 0, 1, \ldots, \ell - 1 \) and grouping them yields

\[
\bar{Y} = \bar{V}V
\]

(6)

where

\[
\bar{Y} = \begin{bmatrix} \beta_0 & -\alpha_1 & \beta_1 & -\alpha_2 & \beta_2 & -\alpha_3 & \cdots & \beta_p & -\alpha_p \end{bmatrix}
\]

(7)

The solution for \( \bar{Y} \) containing the OMP is obtained by solving the equation

\[
\bar{Y} = \bar{Y}V\dagger
\]
be taken instead. Otherwise, a pseudoinverse must be used. Note that, because the size of $VV^T$ is much smaller than $V$, a pseudoinverse may be appropriate even if the product is well conditioned.

The expanded form of the multistep output prediction equation for the case in which the control horizon is equal to the prediction horizon is shown as follows

$$y_p(k) = T u_p(k) + B u_p(k - p) - A y_p(k - p)$$

where

$$T = \begin{bmatrix}
\beta_0 & 0 & \cdots & 0 \\
\beta_{p-1} & \beta_0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{p-1}^{(p-2)} & \beta_{p-2} & \cdots & \beta_0
\end{bmatrix}$$

$$B = \begin{bmatrix}
\beta_0 & \beta_{p-1} & \cdots & \beta_1 \\
\beta_1 & \beta_0 & \cdots & \beta_1 \\
\vdots & \vdots & \ddots & \vdots \\
\beta_{p-1} & \beta_{p-2} & \cdots & \beta_1
\end{bmatrix}$$

$$A = \begin{bmatrix}
a_0 & a_{p-1} & \cdots & a_1 \\
a_1 & a_{p-1} & \cdots & a_1 \\
\vdots & \vdots & \ddots & \vdots \\
a_{p-1} & a_{p-2} & \cdots & a_1
\end{bmatrix}$$

$$\theta = \theta_0 + \sum_{i=1}^{n_e} y_i \psi_i$$

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where $\theta_0$ is the nominal vector, and $y_i$ is the identified uncertainty coordinates of the identified principal directions $\psi_i$. Note that the total number of elements in $\theta$ is $p m x + pm m + pm x pr = p^2 m (2r + m)$. For a multi-input and multi-output system with a large integer $p$, the number may be very large to deal with for real-time/online estimation of uncertainty quantification. On the other hand, the independent number of parameters is $p m^2 + p + 1) m r$, i.e., the total number of elements in $Y$ defined in Eq. (6). Indeed, one may quantify the uncertainty of the independent parameters in $Y$ rather than those in the parameter vector $\theta$. However, it is much more complicated computationally to quantify the uncertainties of the coefficient matrices $T$, $B$, and $A$ based on the uncertainty quantification of independent parameters because it involves the nonlinear recursive relations shown in Eq. (12). It will be shown later that only a very limited number of identified principal directions $\psi_i$ shown in Eq. (14) is sufficient to quantify the uncertainty of the parameter vector $\theta$ that contains the elements of $T$, $B$, and $A$.

The deviation of each identified parameter vector $(n_e$ experimental tests or different time periods of data) from the nominal vector is calculated by

$$\Delta \theta_i = \theta_i - \theta_0, \quad \theta_0 = \frac{1}{n_e} \sum_{i=1}^{n_e} \theta_i; \quad i = 1, \ldots, n_e$$

An uncertainty matrix of $n_e \times n_e$ is defined by

$$\Delta \Theta = [\Delta \theta_1 \Delta \theta_2 \cdots \Delta \theta_{n_e}]$$

The weighted uncertainty matrix is computed as

$$\Delta \hat{\Theta} = W^{-1} \Delta \Theta$$

where $W$ is a diagonal matrix to normalize parameter variations.

Computing the SVD of $\Delta \hat{\Theta}$ yields

$$\Delta \hat{\Theta} = \tilde{U} \tilde{S} \tilde{V}^T$$

$$\tilde{S} = \text{diag}(s_1 \ldots s_n)$$

where the matrices $\tilde{U}$ and $\tilde{V}$ are orthonormal matrices, i.e., $\tilde{U}^T \tilde{U} = \tilde{V}^T \tilde{V} = \mathbf{I}_n$ (identity matrix of order $n$). The $n_e \times n$ basis matrix $U$ for $\Delta \Theta$ is calculated by

$$U = \tilde{W} \tilde{U}$$

$$U = [\psi_1 \cdots \psi_n]$$

If we assume that $n_e \geq n$, then $n_e > n$, i.e., the number of nonzero singular values is at least one less than the number of experiments due to the subtraction of the mean value shown in Eq. (15) for computing the error matrix $\Delta \Theta$. In view of Eqs. (17) and (18), the coordinate matrix $\Gamma$ of $\Delta \Theta$ corresponding to the basis matrix $U$ is produced by

$$\Delta \Theta = UT \Rightarrow \Gamma = \tilde{U}^T W^{-1} \Delta \Theta = \tilde{U}^T \Delta \hat{\Theta} = \tilde{S} \tilde{V}^T$$

which, in turn, yields the estimated parameter vector $\hat{\theta}_i$ for the $i$th experiment

$$\hat{\theta}_i = \hat{\theta}_0 + \sum_{j=1}^{n_e} \Gamma_{ij} \psi_j; \quad i = 1, 2, \ldots, n_e$$

III. Uncertainty Quantification

The coefficient matrices $T$, $B$, and $A$, describing the input-output map of the system, are subject to system/measurement uncertainties. Let $\theta$ be a parameter vector containing all elements of the coefficient matrices $T$, $B$, and $A$, i.e.
As a result, the variance of the coordinate $y_j$ defined in Eq. (14) can be estimated by the following equation

$$\sigma_i^2 = \frac{1}{n_x - 1} \sum_{i=1}^{n_x} \left( \Gamma_{ji} - \frac{1}{n_x} \sum_{k=1}^{n_x} \Gamma_{jk} \right)^2 ; \quad j = 1, 2, \ldots, n \quad (22)$$

The standard deviation $\sigma_i$ represents the distribution of uncertainty in the direction of the principal direction $\psi_j$. Here, we will investigate the relationship between standard deviation $\sigma_i$ and singular value $s_i$. From Eqs. (15) and (20), the mean vector of the column vectors of the coordinate matrix $\Gamma$ can be computed as

$$\frac{1}{n_x} \sum_{i=1}^{n_x} \left[ \begin{array}{c} \Gamma_{1i} \\ \vdots \\ \Gamma_{ni} \end{array} \right] = \frac{1}{n_x} \sum_{i=1}^{n_x} \tilde{U}^T \tilde{W}^{-1} \Delta \theta_i$$

$$= \tilde{U}^T \tilde{W}^{-1} \left[ \left( \frac{1}{n_x} \sum_{i=1}^{n_x} \theta_i \right) - \theta_0 \right] = 0_{n \times 1} \quad (23)$$

Then, the variance of the $j$th uncertainty coordinate can be computed as

$$\sigma_j^2 = \frac{1}{n_x - 1} \sum_{i=1}^{n_x} \Gamma_{ji} = \frac{1}{n_x - 1} \left[ (\Gamma_{ji} \ldots \Gamma_{j0}) \ldots (\Gamma_{j0} \ldots \Gamma_{j0}) \right]^T$$

$$= \frac{1}{n_x - 1} \left[ (0 \ldots 0 s_j 0 \ldots 0) \tilde{V}^T \times \tilde{V} [0 \ldots 0 s_j 0 \ldots 0] \right] = \frac{1}{n_x - 1} s_j^2 \quad (24)$$

As a result, standard deviations can be computed from singular values as

$$\sigma_j = \frac{s_j}{\sqrt{n_x - 1}} \quad (25)$$

All the basis vectors, coordinates, and standard deviations are normalized to the first standard deviation $[14]$. The parameter vector $\theta$ with uncertainty is then quantified as

$$\theta = \theta_0 + \sum_{i=1}^{n} \gamma_i \psi_i \quad (26)$$

where the integer $n$ may be considerably smaller than $n_x$ (number of parameters), the first identified uncertainty coordinate $\gamma_i$ is $\sigma_i$, and the first normalized standard deviation $\sigma_i$ is 1. From Eq. (25), the normalized standard deviation $\sigma_j/\sigma_1$ is the same as the normalized singular value $s_j/s_1$, and they can be used to determine the number of dominant uncertainty coordinates.

IV. Optimal Controller Design

Consider the output prediction model described by Eq. (8) and the parameter vector $\theta$ defined in Eq. (13) with uncertainty quantified by Eq. (26). The coefficient matrices with the quantified uncertainties become

$$[T \quad B \quad A] = \left[ T_0 + \sum_{i=1}^{n} \gamma_i T_i \quad B_0 + \sum_{i=1}^{n} \gamma_i B_i \quad A_0 + \sum_{i=1}^{n} \gamma_i A_i \right] \quad (27)$$

where the quantities

$$[T_0 \quad B_0 \quad A_0] \quad \text{and} \quad [T_i \quad B_i \quad A_i] \quad (28)$$

are obtained from the parameter vector $\theta$ defined in Eq. (13) that is computed in Eq. (26). The prediction error between the desired response $y_d(k)$ and the predicted response $y_p(k)$ of the nominal system is defined as

$$\varepsilon_0(k) = y_d(k) - y_p(k) = y_d(k) - T_0 u_p(k) - B_0 u_p(k - p) + A_0 y_p(k - p) \quad (29)$$

The uncertainty error due to the $i$th uncertainty coordinate $\gamma_i$ with the standard deviation $\sigma_i$ is computed as

$$\Delta \varepsilon_i(k) = -\sigma_i [T_0 u_p(k) + B_0 u_p(k - p) - A_0 y_p(k - p)]$$

$$\Delta = -\sigma_i \gamma_i \quad (30)$$

Now, define an objective function $J$ in the errors and controls as

$$J(k) = e_0^T(k)Q_0 e_0(k) + u_p^T(k)R_p u_p(k) + \sum_{i=1}^{n} \sigma_i^2 \gamma_i \gamma_i Q_i y_p(k) + \sum_{i=1}^{n} \sigma_i^2 \gamma_i \gamma_i B_i u_p(k - p) - A_0 y_p(k - p)] \quad (31)$$

Minimizing $J(k)$ with respect to $u_p(k)$ yields

$$u_p(k) = -\left( \tilde{T}_0^T Q_0 T_0 + R + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i T_i \right)^T \tilde{T}_0^T Q [-y_d(k)$$

$$+ \tilde{B}_0 u_p(k - p) - A_0 y_p(k - p)] + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i [B_i u_p(k - p) - A_0 y_p(k - p)] \quad (32)$$

Retaining the first component (first $r$ rows) of $u_p(k)$ produces

$$u_c(k) = \gamma^c y_d(k) - \beta^c u_p(k - p) + \alpha^c y_p(k - p) \quad (33)$$

where

$$\gamma^c = \text{First } r \text{ rows of } \left( \tilde{T}_0^T Q T_0 + R + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i T_i \right)^T \tilde{T}_0^T Q$$

$$\beta^c = \text{First } r \text{ rows of } \left( \tilde{T}_0^T Q T_0 + R + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i T_i \right)^T \times \left( \tilde{T}_0^T Q \tilde{B}_0 + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i \tilde{B}_i \right)$$

$$\alpha^c = \text{First } r \text{ rows of } \left( \tilde{T}_0^T Q T_0 + R + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i T_i \right)^T \times \left( \tilde{T}_0^T Q \tilde{A}_0 + \sum_{i=1}^{n} \sigma_i^2 T_i^T Q_i \tilde{A}_i \right) \quad (34)$$

This is the control law that uses $p$ sets of the preceding input and output measurements.

V. Numerical Example

The BACT system consists of a rigid wing section and a flexible mounting system, as shown in Fig. 2. The detailed description of the BACT model can be found in Waszak [15]. Accelerometers, used as the primary sensors for feedback control, are located at each corner of the wing.
A simplified state-space model was generated for the dynamics of the BACT wind-tunnel model [8]. The open-loop BACT model has two modes, where the second mode becomes unstable when the dynamic pressure is above 150.8 psf at a Mach number of 0.77. Figure 3 shows the magnitude of a transfer function, corresponding to the leading edge inboard acceleration measurement with trailing edge deflection control surface for a Mach number of 0.77 with various dynamic pressures.

The model used in this study has two inputs, the trailing edge deflection control surface and the upper spoiler deflection control surface, and two outputs, the trailing edge inboard acceleration and the leading edge inboard acceleration. The study in this paper is based on the simulation of the BACT analytical model with the assumed time-varying dynamic pressure and Mach number. For system identification, the real-time random input response data are generated from this BACT model with one added sine-wave disturbance input of amplitude 0.1 at a frequency of 6 Hz plus the added measurement noise, normally distributed white noise with zero mean. Both external disturbance and measurement noise are treated as unknown. The dynamic pressure of the BACT model increases with time, starting from 125 psf with an increment rate of 60 psf/min, whereas the Mach number decreases with time, starting from 0.85 with a reduction rate of 0.18/min. Figures 4 and 5 show the poles and damping ratios of the two modes of 21 BACT models from the zeroth second to the 200th second at increments of 10 s, whereas the dynamic pressure increases from 125 psf to 325 psf and the Mach number decreases from 0.85 to 0.25. Note that the damping ratio of the first mode increases with time, and it is always positive; however, the damping ratio of the second mode decreases with time, and it becomes negative (see Fig. 5) at around the 25th second with dynamic pressure 150 psf and Mach number 0.775.

The input/output data are generated at a sampling rate of 50 samples/s. A least-squares technique [5] is applied to synthesize each block of input/output data with 150 samples to produce the identified output prediction matrices, T, A, and B. The first identified model is based on the data from the zeroth second to the third second. After the first model is identified, the model is updated every second. 15 identified models are used for uncertainty quantification, where the last model corresponds to the data from the 14th second to the 17th second. The computational time for identifying these 15 models is 0.063 s, so each model is identified within 0.005 s. The model order \( p = 5 \) in Eq. (1) is chosen. The size of each output prediction matrix, i.e., \( T, A, \) or \( B \), is \( 10 \times 10 \) with 100 elements. There are a total of 300 elements for three output prediction matrices, \( T, A, \) and \( B \). Figure 6 shows four identified elements of the 15 identified models, where dynamic pressure changes from 125 psf to 142 psf within 17 s. These four elements are \( T_{11}, T_{12}, T_{14}, \) and \( B_{11} \). The elements \( T_{11}, T_{12}, \) and \( T_{14} \) vary linearly with time, whereas element \( T_{14} \), which has the smallest magnitude, varies randomly with time.

The preceding uncertainty quantification technique is used to synthesize the 300 identified elements of the 15 identified models. Figure 7 shows the standard deviations of the identified uncertainty coordinates and the singular values normalized to the first

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**Fig. 3** Transfer function for Mach = 0.77, dynamic pressure = 130, 140, 150, 160, 170 psf.

**Fig. 4** Poles of models with various flight conditions: * (first mode), ○ (second mode).

**Fig. 5** Damping ratio as function of flight time. a) first mode and b) second mode.

**Fig. 6** Variation of four identified elements from 3 s to 17 s: a) \( T_{11} \), b) \( T_{12} \), c) \( T_{14} \), and d) \( B_{11} \).
one, where the 15th standard deviation and the 15th singular value are zero. The standard deviations are identical to the normalized singular values.

Figure 8 shows the acceleration of the first output of the open-loop system in the first 40 s with the dynamic pressure increasing from 125 psf to 165 psf. After 25 s, the system becomes unstable at dynamic pressure 150 psf and Mach number 0.775.

After the uncertainty quantification, the uncertainty error is included in the design of predictive control. In this study, the weighting matrix $Q$ for the prediction-error term is chosen as an identity matrix. The control weighting matrices $R$ (control force) and $Q_k$ (uncertainty error) are assumed to be diagonal with $w_1$ and $w_2$ as the diagonal elements, respectively. The controller is designed based on the identified coefficient matrices with the quantified uncertainty in Eq. (27) computed from the 15 identified models within 17 s. The computational time for uncertainty quantification is 0.031 s, and the computational time for the controller design is 0.015 s, where the simulation is conducted using a Dell mini tower desktop and MATLAB software. The controller is turned on at the 18th second, 1 s after the last identified model used for uncertainty quantification. The control gain matrices in Eq. (34) are fixed. Figure 9 shows the results of the closed-loop systems when the weight $w_1$ for the control force is 0.03. For the design with $w_2 = 0$, which corresponds to the predictive controller without including the uncertainty error, the closed-loop system becomes unstable at around the 35th second with dynamic pressure 160 psf and Mach number 0.745. However, the designed controller with $w_2 = 10000$, which weights the uncertainty error for a predictive control design, stabilizes the system within 80 s with dynamic pressure reaching 205 psf and Mach number reducing to 0.61. Results of the closed-loop design with four uncertainty coordinates ($n = 4$) are close to those with eight uncertainty coordinates ($n = 8$) because the first four uncertainty coordinates characterize and capture the major uncertainty, and the cost function related to the $j$th uncertainty coordinate is weighted with $\sigma_j^2$.

Figure 10 shows the trailing edge control input between the 17th second and the 20th second for the design with $n = 8$ when the controller is turned on at the 18th second. Little feedback force is contributed from the prediction-error term because relatively large $w_1 (=0.03)$ gives small control force. The robustness term related to the quantified uncertainty tunes the control force and significantly improves the stability of the system, and the major feedback control force is contributed from this part. Recall that the total input force $u$ is the summation of the generated random input signal $u_{id}$ and feedback control force $u_c$, as shown in Fig. 1.

Figures 11–13 show the results for the predictive control design including prediction and uncertainty errors when the weight $w_1$ for the control force is 0.0005 with $n = 8$, i.e., eight uncertainty coordinates for the predictive control design. The controller is turned on at the 18th second, and the control gain matrices in Eq. (34) are fixed. The poles of this controller are $-0.5988 \pm 0.2171i$, $-0.3381 \pm 0.5175i$, $0.0568 \pm 0.6990i$, $0.6661 \pm 0.1783i$, and $0.5003 \pm 0.3162i$, respectively. The absolute values of these poles are less
increases from 125 psf to 345 psf, and the Mach number reduces from 0.85 to 0.19. A time-domain system identification technique, such as observer/Kalman filter identification (OKID) [16], is applied to synthesize the collected input/output data of the closed-loop system to update the models of the close-loop system within 220 s. The identified closed-loop model is updated every second, and it is always stable within 220 s. Figure 12 shows that the designed controller produces excellent vibration suppression after the controller is turned on at the 18th second. This control design gives good performance of robustness and vibration suppression. Figure 13 shows that the feedback control force contributed from the prediction-error term is dominant, whereas the feedback control force contributed from the uncertainty-error term is negligible, and the design including uncertainty gives little robustness improvement.

For comparison, we also investigated several cases where no external periodic disturbance and measurement noise were added to the BACT model. In other words, uncertainty quantification was performed only for the BACT model alone having time-varying dynamic pressure and Mach number. No significant difference for the closed-loop results was found for the BACT model with or without unknown external disturbance and measurement noise. This is consistent with the earlier research results presented in Juang and Eure [4], where GPC was designed to deal with unknown disturbance for time-invariant linear systems.

VI. Conclusions

This paper presents an innovative RGPC method, which is successfully applied for active flutter suppression under changing flight conditions. In this approach, uncertainty quantified directly from measured data is used to design the controller parameters. The proposed method provides an innovative and efficient way to incorporate quantified uncertainty into the predictive control design for robustness performance. The results of the BACT example demonstrate that the proposed method requires insignificant computational efforts for system identification, uncertainty quantification, and control design. With the use of data from a wind-tunnel model, the example demonstrates that the developed uncertainty quantification technique is able to extract and characterize the dominant uncertainty coordinates from a system with a large number of identified parameters. The standard deviations of the identified uncertainty coordinates cannot only be used to determine the number of dominant uncertainty coordinates but can also be used to weight the uncertainty error for robust predictive control. It is shown that the proposed technique can significantly improve the robustness performance for flutter vibration suppression of a wind-tunnel model when small control force is applied. Indeed, the proposed robust predictive control approach with uncertainty quantification provides an efficient and effective tool for problems, such as active aircraft flutter suppression under flight condition variations.

References


