GUARANTEE APPROACH
FOR ORBIT DETERMINATION

Z.Khutorovsky∗, V. Boikov†, S. Sukhanov‡, S.Kamensky§,
N.Sbytov¶, A. Samotokhin∥, T.Alfriend∗∗

We consider the guarantee approach task setting and describe the three most interesting procedures for solving the problem. Not optimal and the most simple one is applied for the problem of orbit determination on the basis of radar measurements for one pass through the radar. The mathematical simulation demonstrates that, for the errors of the measurements with no time correlation and uniform distribution, the guarantee approach estimates, of the most rough parameters of position and velocity vector in the local spherical coordinate frame, on average are less (about 3 times) than the errors of the least squares estimate errors.

Introduction

For the time correlated errors of single measurements††, and also for all the cases when the statistical characteristics of these measurements are not known completely, the traditional techniques based on either least squares method (LS) or recurrent Kalman’s filtering do not provide the guarantee evaluation of the accuracy of the obtained estimates.

Now this limitation becomes important since the problem of accurate evaluation of the collision hazard for certain dangerous approaches, requires the compliance between the actual and calculated errors of orbit determination. Thus, there is a certain demand for the guarantee approach (GA) for orbit determination on the basis of measurements.

The GA based estimates of the orbital parameters must satisfy two basic conditions:

1. Along with the estimates of the parameters, the guarantee ranges of the errors for each of them must be provided.

2. For the cases of efficiency∗ of traditional methods† the accuracy of GA estimates should not be lower than the accuracy of traditional methods‡.

---

∗Doctor technical science, Russia, "Vympel" Corporation, manager section (ballistics and navigation
†Ph.D., Russia, "Vympel" Corporation, lead scientist
‡Doctor technical science, Russia, "Vympel" Corporation, Designer general
§Doctor technical science, Russia, "Vympel" Corporation, Chief designer
¶Russia, "Vympel" Corporation, lead programmer
∥Russia, KIAM, senior scientist
∗∗TEES Distinguished Research Chair Professor, USA, Texas A&M University
††for the radar – measurements acquired by one pulse, for optical sensor - one "mark" of angular coordinates
*and even more for the cases of non-efficiency
†linearized problem, not correlated errors of measurements
‡for the distribution of the errors close to the distributions of actual errors
The only assumption for which the task should be solved is that there are known upper limit constants for the errors of the measurements. No other assumptions regarding these errors (probability distribution, correlation characteristics) are used. The approach for the search and optimization of the guarantee ranges for the estimates of the parameters for the first time arrived in the 60-s [1]. Then it has been developed in many works (see, for example, [2,3]) and by early 80-s a rather complete theory of these estimates was created that used the mathematical tools of the functional analysis in normalized linear spaces [11].

For the tasks related to the orbit determination on the basis of measurements the GA first has been used by M.L. Lidov and P.E. Eliasberg [4,5]. They (and their successors) were looking for the guarantee estimates within the class of LS estimates that provided minimum errors for the least favorite distribution of the errors of the measurements. It was demonstrated that for this case, the best way is to use minimum number of measurements with the optimal selection of these measurements from the available set based on solving a certain linear programming problem. Such an estimate satisfies the condition 1., but however, does not satisfy the condition 2..

The GA in the 70-80-s was widely applied in different areas (power engineering, electronics, chemistry, biology etc.). However, in space information systems (in particular in the Russian space surveillance system) this approach was not used. There were certain attempts [6,7], but they did not reach the level of actual tools for the space information systems that satisfied the conditions 1. and 2. There was a reason for this. The case is that the GA procedures require significantly greater computer capacity than the LS techniques. This was a important limitation for the 60-70-s. Now the computer capacities increased by orders of magnitude and the situation is different.

Why is there no new interest for these techniques now ? There are many subjective reasons for this. The major one is the fact that all the systems focused on processing of real measurements of space targets are already created. Under this condition the best assumed way for improvements is the development of new sensors and modernization of the existing ones with the idea of having more measurements available. In addition, many specialists adhere to the position that "nothing can be better than the LS", since the LS track is the closest to the existing measurements. How can we fit them better ?

Thus, prior to starting the development of new sophisticated procedures based on GA, we should convince ourselves that there are important practical problems of orbit determination for which we can develop the GA algorithm satisfying the conditions 1. and 2.. The point is that important practical problems are of interest, and not model examples. The work [8] uses the very simple model example for construction of a GA estimate and demonstrates that for certain distributions of the errors of measurements, its accuracy is essentially higher than that of the LS estimate. However, this result only stimulates further research and in no way proves the efficiency (meaning the conditions 1. and 2.) of the GA for orbit determination using measurements. We have to prove the "theorem of existence" for the practical task for which the GA is efficient. This paper makes an attempt to do this.

**Task setting and algorithms**

Let for the times \( t_k \ (t_k \leq t_{k+1}; \ k = 1, 2...n) \) the measurements \( u_k \) of certain
functions \( h_k(\mathbf{a}) \) of a \( m \)-dimensional vector of parameters are acquired \( \mathbf{a}^* \) \((m \leq n)\), and the errors of the measurements \( \delta_k = u_k - h_k(\mathbf{a}) \) have upper limits of known constants \( \Delta_k \), i.e.

\[ u_k = h_k(\mathbf{a}) + \delta_k \quad |\delta_k| \leq \Delta_k \quad k = 1, 2, \ldots, n \quad (1) \]

The inequalities \(|u_k - h_k(\mathbf{a})| \leq \Delta_k \) \((k=1,2,\ldots,n)\) determine in the \( m \)-dimensional space of parameters \( \mathbf{a} = (a_1, a_2, \ldots, a_m) \) a domain \( \mathbf{D} \) of possible values of \( \mathbf{a} \)

\[ \mathbf{D} = \bigcap_{k=1}^{n} \mathbf{D}_k \quad \mathbf{D}_k = \{ \mathbf{a} : u_k - \Delta_k \leq h_k(\mathbf{a}) \leq u_k + \Delta_k \} \quad (2) \]

If \( \mathbf{D} \) is not empty, any of its points can be an acceptable estimate of the unknown parameter \( \mathbf{a} \), and the "size" of \( \mathbf{D} \) is the measure of parametric uncertainty, i.e. determines the maximum errors of this estimate.

We will consider the following three techniques among the possible ways to determine such an estimate:

1. central estimate \( \mathbf{a}_c \),
2. projected (tightened) estimate \( \mathbf{a}_p \),
3. group estimate \( \mathbf{a}_g \).

**Central estimate \( \mathbf{a}_c \).** The domain \( \mathbf{D} \) is projected to the coordinate axes of the components of vector \( \mathbf{a} \), and among the projected points on the \( i \)-th axis \((i=1,2,\ldots,p)\) the most left \( a_{i,m} \) and the most right \( a_{i,M} \) points are determined. They define the boundaries (maximum and minimum values) of the interval of variation of each of the components of parameter \( \mathbf{a} \). The components of vector \( \mathbf{a}_c \) are the mean points of these intervals, and the half-lengths of the intervals are the maximum values of the errors

\[
\begin{align*}
    a_{i,m} &= \min_{\mathbf{a} \in \mathbf{D}} a_i \\
    a_{i,M} &= \max_{\mathbf{a} \in \mathbf{D}} a_i \\
    \mathbf{a}_m &= (a_{1,m}, a_{2,m}, \ldots, a_{m,m}) \\
    \mathbf{a}_M &= (a_{1,M}, a_{2,M}, \ldots, a_{m,M}) \\
    \mathbf{a}_c &= 0.5(\mathbf{a}_M + \mathbf{a}_m) \\
    \Delta \mathbf{a}_c &= 0.5(\mathbf{a}_M - \mathbf{a}_m)
\end{align*}
\quad (3)
\]

This estimate possesses some optimal properties that are formulated in the terms of normalized spaces. Under the condition (1) the most natural length (norm) of the vector \( \mathbf{x} = (x_1, x_2, \ldots, x_r) \) in \( r \)-dimensional Euclidean space is not its common length \( \sqrt{x_1^2 + x_2^2 + \ldots + x_r^2} \), but its maximum component \( \max_i |x_i|^1 \). For this way of determination of the "distance" in the \( r \)-dimensional space the central estimate\(^\dagger\) is the center of the minimum sphere\(^\S\), containing the set \( \mathbf{D} \). This estimate in the class of correct estimates of parameters \(^\¶\) provides the minimum of the error\(^\**\) for the least favorable\(^\*\) realization

\(^*\)vectors and matrices are in bold, \(^'\) transposition sign, without the sign \(^'\) vector is considered a column vector
\(^\dagger\)this norm is denoted \( l_\infty \) that differs from the common one denoted \( l_2 \). Both norms are particular cases of the norm \( l_p \), defined as \( \|\mathbf{x}\|_p = \left( \sum_{i=1}^{r} |x_i|^p \right)^{1/p} \)
\(^\S\)it is as well called the Chebyshev center of the set \( \mathbf{D} \)
\(^\¶\)in the space with the metric \( l_\infty \) the sphere is a cube
\(^\**\)providing precise value of parameter for the measurements with no errors
\(^\*\)in the sense of the norm \( l_\infty \)
\(^\**\)in the sense of the norm \( l_\infty \)
of the errors of the measurements and for the least favorable†† location of the unknown parameter in the domain $D$ (see, for example, [9]).

**Projected (tightened) estimate** $a_p^*$. We reduce the values of maximum errors $\Delta_k$, used for the construction of the domain $D$, until this domain exists, and find such minimum values for which $D$ is not empty. Doing this we "tighten" the domain $D$ to a point†. This point is the tightened estimate and the maximum distances from this point to the boundary of $D$ in $i$-th component – are the maximum values of the errors in this component

$$D_\alpha = \bigcap_{k=1}^{n} D_{\alpha,k} = \{a : u_k - \alpha \cdot \Delta_k \leq h_k(a) \leq u_k + \alpha \cdot \Delta_k\}$$

$$a_m = \min_{D_\alpha \neq \emptyset} \alpha_p \in D_{\alpha_m} \Delta a_{p,i} = \max_{a \in D} |a_{p,i} - a_i| \quad i = 1, 2, \ldots, p \quad (4)$$

The tightened estimate is projected (see, for example, [10]), i.e it provides minimum distance between the vector of measurements $u = (u_1, u_2, \ldots, u_n)$ and the vector $h(a) = (h_1(a), h_2(a), \ldots, h_n(a))$‡ for any norm. In particular, for the norm $l_2$ the tightened estimate is the LS estimate, and for the norm $l_1$ - the estimate of the least modules method.

**Group estimate** $a_g$. Consider all possible combinations (groups) of $n$ measurements containing $m$ measurements. For each group $g$ of $m$ measurements calculate the vector of parameters $a_{gr}$, solving the respective system of $m$ equations with $m$ unknown quantities. The domain of uncertainty $D_{gr}$ of the estimate $a_{gr}$ is approximated by a $m$-dimensional parallelepiped with the center in the point $a_{gr}$, which in its turn is approximated by the $m$-dimensional rectangle with $2m$ $m-1$-dimensional faces, parallel to $m$ $m-1$-dimensional coordinate planes. The intersection of all these rectangles for all possible groups of $m$ measurements forms a certain rectangle $R$ with faces, parallel to coordinate planes. Its center will be the group estimate $a_g$, and the half-lengths of the edges of the rectangle $R$, parallel to the coordinate axes, - will be the maximum values of the errors of the components of $a_g$.

Note that for multi-dimensional problems, when different functions of parameter $a$ with essentially different errors for each of them, the volume of the rectangle $R$ often turns out to be greater than the volume of the domain $D$, and, thus, the estimate $a_g$ is more rough than the estimates $a_c$ and $a_t$ (see Fig.1). At the same time the determination of the central and projected estimates even for the linear cases is a sophisticated mathematical problem, and the group estimates, even for non-linear cases, can be found easier. We will come back to this problem in the end of the paper, when considering the practical case of orbit determination by measurements.

†† in the sense of the norm $l_\infty$

*or robust-interpolatory [3,10]

† In particular case this may be a sub-domain with dimension $r<p$

‡ if $h(a) = H \cdot a$, the estimate is the result of projecting the vector $u$ of $n$-dimensional space to the $m$-dimensional space, tightened on the columns of the matrix $H$
Linearization

Assume that

1. we know the value of parameter $a^*$, for which the residuals $u_k - h_k(a^*)$ have the order of the errors of the measurements;

2. within the interval of possible variation of the errors of measurements the function $h_k(a)$ with enough accuracy is a linear function of the shape

$$ h_k(a) = h_k(a^*) + g_k'(a^*) \cdot (a - a^*) \quad g_k(a) = \operatorname{grad}_a h_k(a) \quad k = 1, 2, ..., n \quad (5) $$

Then, the condition (1) takes the form

$$ v_k = g_k' \cdot b + \delta_k \quad |\delta_k| \leq \Delta_k \quad (6) $$

where

$$ v_k = u_k - h_k(a^*) \quad g_k = \operatorname{grad}_a h_k(a^*) \quad b = a - a^* \quad (7) $$

For the $a^*$ we can take, for example, the LS estimate of parameters. The methods for obtaining this estimate are well known. We know as well that the LS estimate is always accurate enough, and the residuals $v_k$ have the order of measurement errors $\delta_k$. However, the correct (not under or over estimated) guarantee evaluations of the accuracy for the LS estimate, in practical cases (our case is not an exception), usually can not be obtained.

Considering the condition (7) the domain $D$ takes the shape of a convex polyhedron

$$ D = \bigcap_{k=1}^{n} D_k \quad D_k = \{ b : v_k - \Delta_k \leq g_k' \cdot b \leq v_k + \Delta_k \} \quad (8) $$

in the $m$-dimensional space of parameters $b$, resulting from the intersection of $n$ bands between two parallel $m-1$-dimensional planes.

When the problem is linearized, the algorithm for obtaining each of the three above mentioned estimates of parameters takes the following form:

1. **Central estimate** $b_c$ and its error $\Delta b_c$ are obtained by formulas

$$ b_c = 0.5 \cdot (b_M + b_m) \quad \Delta b_c = 0.5 \cdot (b_M - b_m) \quad (9) $$

where the components $b_{i,m}$ $b_{i,M}$ of $m$-vectors $b_m$ $b_M$ are the solutions of the following $2 \cdot m$ linear programming problems:

$$ \begin{align*}
\text{find} & \quad b_{i,m} = \min_{b \in D} a_i \\
\text{for the constraints} & \quad v_k - g_k' \cdot b \leq \Delta_k \quad v_k - g_k' \cdot b \geq -\Delta_k \quad k = 1, 2, ..., n
\end{align*} \quad (10) $$

2. **Projected estimate** $b_p$ is obtained by solving the following linear programming problem

$$ \begin{align*}
\text{find} & \quad \alpha_m = \min_{b \in D_a} \alpha \\
\text{for} & \quad b_p = a - \alpha \quad (11)
\end{align*} $$
for the constraints  \( v_k - g'_k \cdot b - \alpha \cdot \Delta_k \leq 0 \)  
\( v_k - g'_k \cdot b + \alpha \cdot \Delta_k \geq 0 \)  
\( k = 1, 2, \ldots, n \)
in the \( m+1 \) dimensional space of parameters \( b \) and \( \alpha \), and its errors (components \( \Delta b_{p,i} \) of \( m \)-vector \( \Delta b \)) - are the solutions of the following \( m \) linear programming problems

find  
\( \Delta b_{p,i} = \max_{b \in D} \{ |b_{p,i} - b_i| \} \)  
\( \text{under the constraints} \)  
\( v_k - g'_k \cdot b \leq \Delta_k \)  
\( v_k - g'_k \cdot b \geq -\Delta_k \)  
\( k = 1, 2, \ldots, n \)
in the \( m \) dimensional space of parameters \( b \),

3. **Group estimate** \( b_g \) and its error \( \Delta b_g \) are obtained using formulas

\[
b_g = 0.5 \cdot (\tilde{b}_M + \tilde{b}_m) \quad \Delta b_g = 0.5 \cdot (\tilde{b}_M - \tilde{b}_m)
\]  
\( \text{similar to (9). Here the boundaries} \) \( \tilde{b}_m \) and \( \tilde{b}_M \) of the rectangle \( R \) are determined as

\[
\tilde{b}_m = \max_g g_m \quad \tilde{b}_M = \min_g g_M
\]

where \( g_m \) \( g_M \) - the boundaries of the rectangle \( R_g \), describing the uncertainty domain \( D_g^* \) of the estimate of the parameters \( b_{g,gr} \), obtained for certain group \( g \), containing \( m \) measurements selected from \( n \) measurements\(^1\).

The boundaries \( g_m \) \( g_M \) for each group \( g \) have the shape

\[
g_m = \min_{\text{vert } D_g} b \quad g_M = \max_{\text{vert } D_g} b
\]

where \( \text{vert } D_g \) are the parameters \( b \) \( 2^m \) of the apexes of the parallelepiped \( D_g \), which are the solutions of \( 2^m \) systems of linear equations of the form

\[
\tilde{v}_{gr} = G'_{gr} \cdot b \quad \tilde{v}_{gr}' = (\tilde{v}_1 \pm \tilde{\Delta}_1, \tilde{v}_2 \pm \tilde{\Delta}_2, \ldots, \tilde{v}_m \pm \tilde{\Delta}_m)
\]

where

\( - G_{gr} - m \times m \) the matrix, "cut" from \( m \times n \) matrix \( G = (g_1, g_2, \ldots, g_n) \) according to \( m \) measurements of a certain group;

\( - \tilde{v}_i, \tilde{\Delta}_i \) - the measurements of the certain group and maximum values of their errors  
\( (i = 1, 2, \ldots, m) \).

**Example**

Let us consider the task of orbit determination using measurements acquired during one penetration of a satellite into the radar field of view.

Assume that the radar for the times \( t_k \) \( (k = 1, 2, \ldots, n) \) measures the range \( d \), azimuth \( \alpha \), and elevation angle \( \beta \) in the local spherical coordinate frame, and the errors of the measurements do not exceed, respectively, the values \( \Delta d, \Delta \alpha, \Delta \beta \). The task is to determine for the time \( t = 0.5 \cdot (t_1 + t_n) \) the six-dimensional \( (m=6) \) vector of orbital

---

\(^*\)in this case the \( m \)-dimensional parallelepiped

\(^1\)the number of these groups is equal to the number of combinations from \( n \) for \( m \) \( C_n^m \), that is \( n!/m!(n-m)! \)
parameters \( \mathbf{a} = (d, \alpha, \beta, \dot{d}, \dot{\alpha}, \dot{\beta}) \) in this coordinate frame and its maximum possible errors \( \Delta \mathbf{a} = (\Delta d, \Delta \alpha, \Delta \beta, \Delta \dot{d}, \Delta \dot{\alpha}, \Delta \dot{\beta}) \).

For solving the problem we use the GA for group estimate of the parameters. The linearization will be performed for the LS estimate calculated using the algorithm described in [8].

The above described algorithm for obtaining the group estimate implies the enumerative search over \( C^6_3 \) different groups of measurements. For typical realistic values of \( n \) \((\approx 10-100)\) this can be a very great number, that makes doubtful the implementation of the algorithm for modern computers. Thus, we will consider only those groups of measurements where all the parameters are present and acquired for the same time. Then only 2 measurements (in this case 3-dimensional) of \( n \) will participate in the search, and the number of possible combinations is \( C^n_2 = n(n-1)/2 \).

The characteristics of this algorithm will be evaluated using a statistical simulation and then they will be compared with the LS results.

The simulation was performed for the following initial conditions:

- **coordinates of radar location**: \( \varphi = 0.7, \lambda = 0, h = 0; \)
- **measurement errors**: the RMS of the non correlated errors of the measured parameters \( \sigma_d = 0.05 \text{km}, \sigma_\alpha = 0.001 \text{rad}, \sigma_\beta = 0.001 \text{rad}; \) there are no correlated and abnormal errors; the distribution of the errors is either uniform or Gaussian†;
- **the radar field of view** is limited for the elevation angle by the minimum value of 5° and the maximum of 60°.
- **satellite trajectories**. Two orbits are chosen, both with 1 radian inclination, the first one with an average altitude above the Earth surface, 800 km, the second one – 1500 km. For each orbit we select two trajectories in the sector of the radar. The first – with "attacking" aspect angle (\( |\dot{\beta}| \gg |\dot{\alpha}| \)), the second – of "transiting" one (\( |\dot{\alpha}| \gg |\dot{\beta}| \)). The Keplerian orbital elements‡ for the first orbit: \( a=7178 \text{km} \ i=57.3^\circ \ \Omega=333^\circ \ e=0.0005 \ \omega=136^\circ \), for the second one – \( a=7878 \text{km} \ i=57.3^\circ \ \Omega=330^\circ \ e=0.0003 \ \omega=124^\circ \).
- **distribution of times of measurements**. We consider that the first measurement is performed in the beginning of the penetration (for the considered trajectories this happens for the minimum elevation angle). Further the measurements are performed each 5 sec. until the satellite leaves the sector (by elevation angle limit).
- **the number of realizations for the simulations** – \( n_r = 100. \)
- **the comparing of the empirical (averaged over the realizations) errors of orbit determination performed using GA and LS approaches** is fulfilled by the end of the tracking interval when the satellite leaves the radar field of view.
- The GA and LS were compared by the empirical mean value \( \bar{\delta} \xi \) of the error calculated for any component \( \xi \) of the vector of orbital parameters \( \mathbf{a} \) using the formula

\[
\bar{\delta} \xi = \frac{\left( \sum_{i=1}^{n_r} |\xi_i - \xi| \right)}{n_r}
\]

where

\( \varphi \) latitude, \( \lambda \) longitude, \( h \) altitude

†limited by the level of \( k \) “sigmas”, \( k=\sqrt{10} \approx 3.2 \)

‡semi-major axis \( a \), inclination \( i \), longitude of the ascending node \( \Omega \), eccentricity \( e \), perigee \( \omega \)
- $\xi$ - the true value of parameter,
- $\xi_i$ - the value, acquired by modeling for the $i$-th realization ($i = 1, 2, ..., n_r$).

The table below presents the results of comparing the empirical average errors of the estimates of parameters $d, \alpha, \beta, \dot{d}, \dot{\alpha}, \dot{\beta}$ for GA and LS for the uniform and Gaussian distributions of the errors of measurements. The LS errors in fact do not depend on the distribution law. Thus, for the LS we give only one value. For the convenience of analysis the angular errors are transferred to linear measure.

<table>
<thead>
<tr>
<th>trajectories</th>
<th>algorithms</th>
<th>$d$ (m)</th>
<th>$\alpha$ (m)</th>
<th>$\beta$ (m)</th>
<th>$d$ (m/s)</th>
<th>$\dot{\alpha}$ (m/s)</th>
<th>$\dot{\beta}$ m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>first</td>
<td>LS</td>
<td>9</td>
<td>160</td>
<td>190</td>
<td>0.08</td>
<td>0.90</td>
<td>0.41</td>
</tr>
<tr>
<td>attacking</td>
<td>GA(u)</td>
<td>25</td>
<td>62</td>
<td>80</td>
<td>0.10</td>
<td>0.74</td>
<td>0.73</td>
</tr>
<tr>
<td>n=65</td>
<td>(g)</td>
<td>34</td>
<td>390</td>
<td>490</td>
<td>0.16</td>
<td>1.40</td>
<td>2.50</td>
</tr>
<tr>
<td>first</td>
<td>LS</td>
<td>10</td>
<td>240</td>
<td>280</td>
<td>0.08</td>
<td>0.40</td>
<td>3.70</td>
</tr>
<tr>
<td>transiting</td>
<td>GA(u)</td>
<td>28</td>
<td>110</td>
<td>130</td>
<td>0.12</td>
<td>2.10</td>
<td>1.90</td>
</tr>
<tr>
<td>n=60</td>
<td>GA(g)</td>
<td>31</td>
<td>480</td>
<td>610</td>
<td>0.18</td>
<td>3.60</td>
<td>7.00</td>
</tr>
<tr>
<td>second</td>
<td>LS</td>
<td>10</td>
<td>200</td>
<td>230</td>
<td>0.04</td>
<td>0.95</td>
<td>0.25</td>
</tr>
<tr>
<td>attacking</td>
<td>GA(u)</td>
<td>22</td>
<td>60</td>
<td>70</td>
<td>0.05</td>
<td>0.60</td>
<td>0.90</td>
</tr>
<tr>
<td>n=100</td>
<td>GA(g)</td>
<td>35</td>
<td>500</td>
<td>570</td>
<td>0.10</td>
<td>1.50</td>
<td>3.20</td>
</tr>
<tr>
<td>second</td>
<td>LS</td>
<td>16</td>
<td>220</td>
<td>260</td>
<td>0.04</td>
<td>0.12</td>
<td>0.73</td>
</tr>
<tr>
<td>transiting</td>
<td>GA(u)</td>
<td>35</td>
<td>130</td>
<td>150</td>
<td>0.07</td>
<td>0.39</td>
<td>0.68</td>
</tr>
<tr>
<td>n=155</td>
<td>GA(g)</td>
<td>48</td>
<td>650</td>
<td>810</td>
<td>0.14</td>
<td>1.80</td>
<td>2.40</td>
</tr>
</tbody>
</table>

The following conclusions follow from the table:

1. The errors of the parameters obtained by GA significantly depend on the distribution of the measurements. For the uniform distribution the errors are less than for the Gaussian law (up to 8 times).

2. For the Gaussian distribution of measurement errors the LS provides more accurate estimates than the GA for all orbital parameters.

3. For the uniform distribution of measurement errors for the worst determined component of position and velocity vectors* for all the considered trajectories, the GA is more accurate than the LS. The maximum benefit of GA compared to LS reaches 3 times. We should note here that for many radars (in particular for all Russian detection radars), the distribution of the actual measurements is closer to the uniform one than to the Gaussian.

4. For the uniform distribution of measurement errors for the most accurate component of position and velocity vectors† for all the considered tracks, the LS is more accurate than the GA.

The last effect is likely to have the following reason. The calculation of the group estimate includes the approximation of the uncertainty domain $D$ by the domain resulting from the intersection of rectangles $R_g$ for all minimum sufficient groups of measurements, and each of these rectangles of the group in its turn is the approximation of the uncertainty parallelepiped of the group. For the case when the measurements have significantly different accuracies‡, such approximations result in an essential extension of the volume.

---

*one of angular coordinates and velocities, in the table indicated by bold font
†range and range rate
‡in our case the range is measured with accuracy of an order of magnitude higher than the angles
of the approximated domain $D$. This produce losses for the most accurate components of vector $a$. For our case these are the range and range rate.

To avoid this we should use more accurate estimates – the central and tightened ones. Our plans include exercising this.

The fulfilled research leads to the very important conclusion: **The GA used for real tasks of orbit determination can provide estimates of orbital parameters, which for certain distributions of the errors of the measurements, close to actual ones, have better accuracy characteristics than the LS estimates.**

### References


5. Eliasberg P.E., Bakhshian B.Ts., Determination of the spacecraft trajectory when knowledge of the errors distribution is lacking, *Space research*, 7, 1, 1969, pp. 18-27.

6. Shekhovtsev A.I., Method for observations processing when the information on the errors distribution is limited, *Spacecraft motion determination*, Nauka, Moscow, 1975, pp. 131. (in Russian)


Fig 1. Central and group estimations.