DETECTION OF UNKNOWN LEO SATELLITES USING RADAR MEASUREMENTS

Alexander S. Samotokhin
Zakhary N. Khutorovsky
Sergey Yu. Kamensky
Russia, Moscow, Vympel Corporation

Terry K. Alfriend
USA, Texas A&M University, College Station
OUTLINE

- INTRODUCTION

- DETECTION ALGORITHM

- DETECTION AND TRACKING ALGORITHM

- SIMULATION OF MEASUREMENTS FOR THE CASE OF BREAK-UP

- RESULTS OF SIMULATION

- CONCLUSIONS
INTRODUCTION

• In the course of processing of radar data with the aim of satellite catalog maintenance a share of measurements does not correlate with cataloged and tracked satellites. These measurements can be used for detection (primary orbit determination) of new unknown objects. The theoretical foundations and the description of the algorithm for solving this task used by Russian Space Surveillance System (SSS) are presented in (Refs. 1, 2). Some practical results are presented in (Ref. 3).

• The orbit detection (primary orbit determination) problems are still a subject of interest for the specialists dealing with processing of significant data fluxes in real time. The recent collision of Kosmos-2251 and Iridium-33 spacecrafts which generated more than 1300 fragments observed by US and Russian space surveillance networks gave additional impulse of interest to this problem.

• The break-ups of tracked satellites are the most severe challenge for orbit detection procedures which most explicitly reveal their features. The most interesting are the characteristics of time efficiency and the reliability of primary determined orbits. The paper suggests to evaluate these characteristics using mathematical simulation of the processes of satellite break-up and the acquisition of radar measurements produced by the fragments.
DETECTION ALGORITHM

TASK SETTING AND THE USED METHOD

- The measurement: the six-dimensional vector of positions and velocities in the radar coordinate frame range, azimuth, elevation angle (RCF)

- The detection condition: the primary orbit determination on the basis of non-correlated measurements provides the accuracy sufficient for reliable correlation of future measurements

- The complete group of non-correlated measurements: all the measurements of the group belong to one satellite and the orbit generated on their basis satisfies the detection condition.

- One and two measurements do not constitute a complete group. The minimum possible size of the complete group of radar measurements in not less than three.

- The primary orbit determination algorithm looks for three non-correlated measurements for different revolutions for which we can find the orbit inscribing into them.
Being part of the catalog maintenance software complex the primary orbit determination code runs periodically in real time scale. The set of non-correlated measurements \( M_{no} \) can be divided into two parts: \( M_{no}^{old} \) and \( M_{no}^{new} \). The set \( M_{no}^{old} \) comprises previously arrived (old) measurements for which the detection procedure have already run and which were not included into the orbits generated previously. The set \( M_{no}^{new} \) comprises the newly arrived measurements which have not yet participated in the detection process.

Let for the new measurement \( x_{new} \), which will be called the reference one, we decided to run the detection computer code. Then the sequence of data fusion is as follows:

- preliminary selection of triplets containing the measurement \( x_{new} \);
- generation of the primary orbit for the selected triplet;
- selection from \( M_{no} \) the measurements inscribing into generated orbit and updating of this orbit by the selected measurements;
- check of reliability of the updated orbit.
PRELIMINARY SELECTION OF TRIPLETS

the group of the measurement $x_{\text{new}}$:

$|i_{\text{new}}-i|<c_i$, $|T_{\text{new}}-T|<c_T$, $|\Omega+(t_{\text{new}}-t)\dot{\Omega}-\Omega_{\text{new}}|_{\text{mod}} 2\pi <\Delta\Omega$, $|t_{\Omega,\text{new}}-t_{\Omega}-\tilde{N}T|<c_t$

where the parameters with index $\text{new}$ refers to $x_{\text{new}}$, and without the index - to any measurement from $M_{\text{no}}$. $\tilde{N}$ - the number of revolutions between $t_{\Omega,\text{new}}$ and $t_{\Omega}$.

We consider that three measurements cannot belong to one satellite if for any $N\in M_N = \{\tilde{N}+i; i=0, \pm 1, \ldots, \pm \Delta N\}$

$|t_{\Omega}^{(2)}-(P/N)t_{\Omega}^{(1)}-(Q/N)t_{\Omega}^{(3)}|>c_{t\Omega}$,

where

1. $P=([t_{\Omega}^{(3)}-t_{\Omega}^{(2)})/T_{\Omega}]$, $Q=N-P$, $T_{\Omega}=(t_{\Omega}^{(3)}-t_{\Omega}^{(1)})/N$;

2. $c_{t\Omega}$ - selecting strobe, chosen experimentally.

The enumerative search of the triplets is arranged in the following way. The left measurements are selected according to increasing reference time, beginning with the most "remote". For the fixed left measurement we select the middle measurements to provide that all three measurements belong to different revolutions. Finally when the measurements are selected the enumeration of possible values of $N$ is performed. The selected triplet along with the evaluation of the number of revolutions between the boundary measurements is forwarded to the primary orbit determination procedure. If certain three measurements were bond into one orbit with $k$ different values of $N$ (number of revolutions between the boundary measurements) the calculation of the primary orbit is performed $k$ times.
CALCULATION OF THE PRIMARY ORBIT

If the triplet of measurements has passed the preliminary selection we make an attempt to generate a primary orbit. For solving this task we perform minimization of the functional

$$
\Phi(a) = \sum_{p=1}^{3} (x_p - f_p(a))' (K_p)^{-1} (x_p - f_p(a)) + (a_{apr} - a)' P_{apr} (a_{apr} - a),
$$

where

1. $x_1, x_2, x_3$ – measurements of the triplet;
2. $f_p(a)$ - functional dependence of parameters of the $p$-th measurement from parameters $a$ of the satellite;
3. $K_p$ – diagonal correlation matrix of the errors of the $p$-th measurement;
4. $a_{apr}, P_{apr}$ – a priori estimate of orbital parameters and its weighting matrix;
5. "'" and "-1" – signs of transposition and inversion of matrices.
CALCULATION OF THE UPDATED ORBIT

Apart from the found three measurements, $M_{no}$ may contain other measurements inscribing into the generated primary orbit. Therefore the "dredging out" of these measurements from $M_{no}$ is performed. For this purpose we use the algorithm for preliminary correlation of measurements used in the tracking process. The decision on the correlation of the measurement from $M_{no}$ to the primary orbit is based on the residuals of its radar parameters with the primary orbit. The specific decision function is determined by the model of the real errors of the measurements.

For all the available measurements (the three initial ones and other correlated) we perform the calculation of the orbital parameters. As distinct from the procedure used for calculation of the primary orbit by three measurements, here:

- The number of the used measurements may be more than three.
- For initial approximation we take the already generated primary orbit.
- For prediction of motion more accurate algorithm is used (the one used in the tracking process).
- Parameter AMR is updated.
- We perform selection of abnormal components and alien measurements.
RELIABILITY CHECK

Further decisions depend on the reliability of the obtained orbit. The reliability criteria must provide further stable tracking of the satellite. These criteria are universal in the sense that they are used not only for the primary determination of orbits but in the tracking process as well for making decision on the reliability of the updated orbit. Thus they are is the core element of the catalog maintenance process as a whole.

The orbit is reliable in case it is obtained and the set of measurements inscribing into it is complete. The orbit is obtained in case the iterative process of minimization of $\Phi(a)$ used for its calculation converged. The measurement is inscribed into the orbit in case its residuals $\Delta$ in coordinate parameters are smaller than the thresholds $c_\Delta$. The condition of completeness is satisfied in case that the number of inscribed measurements is not less than $c_{obs}$ and the number of revolutions $n_{rev}$, for which we have inscribed measurements is not less than $c_{rev}$. The parameters of the decision function (the thresholds $c_\Delta$, $c_{obs}$ and $c_{rev}$) must provide stable tracking of the satellite in future. They are chosen experimentally.
ORBIT IDENTIFICATION

Before making the decision that we have detected a new satellite we should check that the newly obtained orbit is not the orbit of previously cataloged but lost satellite. This is the task for the algorithm of orbit identification.

The errors of orbital parameters are several orders of magnitude smaller than the maximum errors of the measurements. Thus we have an opportunity to make good decisions for rather long time intervals. The same factor determines the simple structure of the algorithm. The decision is made on the basis of comparing the residuals $\delta a$ in orbital parameters with empirically chosen thresholds. For calculation of the residuals we use special long-time prediction of the orbit of the cataloged object to the reference time of the new orbit.

To reduce the computation burden we perform rough selection of the cataloged objects which for sure can not be identified with the new orbit, using parameters $i, \Omega, T$. Identification of any new orbit is performed with all the cataloged objects, both tracked and lost.

When the decision on identification is made the identified cataloged satellite is returned to the tracking process with renewal of orbital parameters. The orbits which are not identified are included into the catalog as new satellites which further participate in the tracking process.
DETECTION AND TRACKING ALGORITHM

The measurements (in reality and in the course of simulation) are processed by the detection and tracking procedure with the time step $dt$. First we process the measurements acquired in the course of the interval $(t_0, t_0+dt)$, Then within the interval $(t_0+dt, t_0 + 2dt)$, etc..

The input measurement is correlated with the already determined and tracked satellites. The conditions of preliminary correlation of the measurement have the shape:

$$|\delta r_v| < c_{r_v} \quad |\delta b_1| < c_{b_1} \quad |\delta b_2| < c_{b_2},$$

where

1. $\delta r_v, \delta b_1, \delta b_2$ - the residuals of coordinate parameters of the measurements with the orbit of the tracked satellite along the direction of the motion and two orthogonal (lateral) directions,

2. $c_{r_v}, c_{b_1}, c_{b_2}$ - the strobes determined experimentally.

If the measurement was preliminary correlated to several tracked satellites it is attributed to the satellite with minimum value of functional

$$f = |\delta r_v| + |\delta b_1| + |\delta b_2|$$
Non-correlated measurements are forwarded to the detection process.

In case the measurement is correlated to certain satellite we try to update its orbit. We use the same algorithm that is used for updating the primarily determined orbit by the primary orbit determination algorithm.

The resulting orbit is tested for reliability. The criteria in the whole coincides with the criteria described for the primary determination algorithm with one addition: the new measurement should be inscribed into the determined orbit.

If the updated orbit is reliable we select the alien and abnormal measurements. The criteria in the whole is similar to the criteria described for the detection algorithm with one addition, however: the decisions are made only for the measurements which have at least one measurement with greater time which is inscribed into the orbit.

The alien measurements are correlated to all the tracked satellites excluding the satellite from which they have been just now selected as alien.
SIMULATION OF MEASUREMENTS FOR THE CASE OF BREAK-UP

The input data for the simulation are:

1. \( n_r \) – the number of radars for which the measurements are simulated.
2. \( \lambda_m, \varphi_m, h_m \) – coordinates of the \( m \)-th radar (longitude, latitude, altitude above the Earth surface) \( m = 1, 2, \ldots, n_r \). The index \( m \) of the number of the radar further is omitted for simplicity.
3. \( A_0 \) – azimuth of the main radiation direction.
4. \( d_{\min}, d_{\max}, \varepsilon_{\min}, \varepsilon_{\max}, \gamma_{\min}, \gamma_{\max} \) – boundaries of the field of view of the radar in range, azimuth, elevation angle in the local radar coordinate frame \( d, \varepsilon, \gamma \).
5. \( \Pi \) – energy parameter of the radar.
6. \( t_0 \) – time of satellite break-up.
7. \(a=(\lambda, L, p, q, h, k)\) – six-dimensional vector of orbital parameters of the satellite for the time \(t_0\).

8. \(s\) – area-to-mass ratio of the satellite (AMR).

9. \(\sigma_d, \sigma_\varepsilon, \sigma_\gamma\) – RMS errors of single measurements (marks) of range, azimuth and elevation angle.

10. \(\Delta\) – time interval between the neighboring marks for each satellite.

11. \(\Delta t_{\text{min}}\) – minimum time interval for which the measurement on the satellite can be acquired during one pass.

12. \(t_{\text{end}}\) – time of finishing the simulation.

13. \(n\) – the number of break-up fragments.

14. \(p(\xi, \eta, \Delta v, s, l)\) – break-up model – distribution of parameters of the break-up with respect to parameters \(\xi, \eta, \Delta v, s, l\), where \(\xi, \eta, \Delta v\) – azimuth, elevation angle and the module of the separation velocity in the orbital coordinate frame \(x_0, y_0, z_0\) of the disintegrated satellite*, \(s, l\) – AMR and the average size of the fragment.
THE SIMULATION ALGORITHM

\[ \Delta \mathbf{v}_{oi} = (\Delta v_i \cos \xi_i \cos \eta_i \quad \Delta v_i \sin \xi_i \cos \eta_i \quad \Delta v_i \sin \eta_i)' \]

\[ \mathbf{r}_e = \mathbf{r}'/r \quad \dot{\mathbf{r}}_e = \dot{\mathbf{r}}'/\dot{r} \quad \mathbf{b}_e = \mathbf{r}_e \times \dot{\mathbf{r}}_e \quad \mathbf{n}_e = \dot{\mathbf{r}}_e \times \mathbf{b}_e \]

\[ \mathbf{E} = (\dot{\mathbf{r}}_e \quad \mathbf{b}_e \quad \mathbf{n}_e) \quad \Delta \mathbf{v}_i = \mathbf{E} \cdot \Delta \mathbf{v}_{oi} \]

\[ \mathbf{r}_i = \mathbf{r} \quad \dot{\mathbf{r}}_i = \dot{\mathbf{r}} + \Delta \mathbf{v}_i' \]

\[ d_{min} < d < d_{max} \quad \varepsilon_{min} < \varepsilon < \varepsilon_{max} \quad \gamma_{min} < \gamma < \gamma_{max} \]

\[ P \cdot (1000/d)^4 \cdot (l^2/0.1) > 1 \]

\[ (d - 5\sigma_d, d + 5\sigma_d) \cap (\varepsilon - 5\sigma, \varepsilon + 5\sigma) \cap (\gamma - 5\sigma, \gamma + 5\sigma) \]

\[ \Delta t > \Delta t_{min}. \]

The precise values of the parameters of the measurements and marks are superimposed with the errors of the measurements:

\[ \tilde{\sigma}_d = \sigma_d \quad \tilde{\sigma}_\varepsilon = \sigma_\varepsilon / \sqrt{n_o} \quad \tilde{\sigma}_\gamma = \sigma_\gamma / \sqrt{n_o} \]

\[ \tilde{\sigma}_d = 3.5\sigma_d/h \quad \tilde{\sigma}_\varepsilon = 3.5\sigma_\varepsilon/h \quad \tilde{\sigma}_\gamma = 3.5\sigma_\gamma/h \]

\[ n_o = \Delta t_{min}/\Delta, \quad h = \sqrt{n_o \cdot \Delta t_{min}}. \]
RESULTS OF SIMULATION

The following initial data have been used for the simulation:

1. The number of radars: \( n_r = 1 \).
2. Radar location: \( \lambda = 0, \ \varphi = 0.5, \ h = 0 \).
3. Energy parameter of the radar: \( \Pi = 27 \).
4. Errors of the measurements: the RMS of the uncorrelated errors of the measured parameters \( \sigma_d = 0.05km, \ \sigma_\alpha = 0.001, \ \sigma_\beta = 0.001 \); no correlated and abnormal errors; normal distribution of the errors;
5. The field of view of the radar has the limits: for the elevation angle by the minimum value \( 1^\circ \) and maximum \( 60^\circ \), for the range \( d \) respectively 100 km and 7000 km.
6. The rate of performing single measurements of the satellite: \( \Delta = 5 \text{s} \).
7. The time interval within which the measurements are performed:
   \( \Delta t_{\text{min}} = 50 \text{s} \).
8. Time interval of simulation of radar measurements: 30 days.
9. The orbit of the break-up satellite: almost circular with inclination \( 65^\circ \) and average altitude above the Earth surface 800 km;
10. The number of break-up fragments: \( n = 300 \);
11. the distribution of the parameters \( \xi, \eta \), of the directions of relative velocities of the fragments: uniform within the intervals \( (0, 2\pi) \) and \( (-\pi/2, \pi/2) \) respectively.
CHARACTERISTICS of the DISTRIBUTION of the PARAMETERS \( l, s, \Delta v \)
of the BREAK-UP FRAGMENTS for "SIGNIFICANT" ("WEAK") DISPERSION

<table>
<thead>
<tr>
<th>sat.class</th>
<th>number of fragments</th>
<th>( l ) (m)</th>
<th>( s ) (m(^2)/kg)</th>
<th>( \Delta v ) (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>3.0 (1.00)</td>
<td>0.003 (0.009)</td>
<td>1 (0.3)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>0.5 (0.17)</td>
<td>0.018 (0.054)</td>
<td>6 (2.0)</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>0.3 (0.10)</td>
<td>0.030 (0.090)</td>
<td>10 (3.3)</td>
</tr>
<tr>
<td>4</td>
<td>80</td>
<td>0.2 (0.07)</td>
<td>0.045 (0.135)</td>
<td>15 (5.0)</td>
</tr>
<tr>
<td>5</td>
<td>200</td>
<td>0.1 (0.03)</td>
<td>0.090 (0.270)</td>
<td>30 (10.0)</td>
</tr>
</tbody>
</table>
PARAMETERS of DETECTION and TRACKING ALGORITHM

1. Strobes for generating groups of non-correlated measurements:
   \( c_t = 0.6^\circ, \ c_n = 0.8^\circ, \ c_T = 3 \text{ min}, \ c_t = 0.3 \text{ min}. \)

2. Strobes for correlation of measurements with tracked satellites (km):
   \( c_{rv} = 5 + 86400 \cdot v \cdot (3 \cdot \sigma_T \cdot n_{rev} + |\Delta T| \cdot n_{rev}^2), \ c_{b_1} = 5, \ c_{b_2} = 5, \) where
   a. \( v \) – the absolute value of the velocity in km/s,
   b. \( \sigma_T \) – RMS of the error of the period calculated from the measurement (days),
   c. \( \Delta T \) – decline of the period by the revolution for the tracked satellite (days),
   d. \( n_{rev} \) – number of revolutions between the times of the measurement and the orbit of tracked satellite.

3. The thresholds for the residual of the measurement with the orbit of the tracked satellite for making decision ”alien measurement”:
   \( 10\sigma_d, \ 5\sigma_\varepsilon, \ 5\sigma_\gamma, \ 5\sigma_\hat{d} \) respectively for the components \( d, \varepsilon, \gamma, \hat{d} \) of the measurement where \( \sigma_d, \ \sigma_\varepsilon, \ \sigma_\gamma, \ \sigma_\hat{d} \) – RMS values for the errors of these components.

4. The thresholds for the residual of the component of the measurement with the orbit of tracked satellite for making decision ”abnormal component”:
   \( 3\sigma \), where \( \sigma \) – RMS value of the error of this component.

5. The strobes for making the decision on the reliability of the orbit generated by the detection algorithm:
   \( c_{\Delta d} = \max (3\sigma_d, 1.0) \), \( c_\varepsilon = 3\sigma_\varepsilon \), \( c_\gamma = 3\sigma_\gamma \),
   \( c_{obs} = 6(7) \), \( c_{rev} = 3(4) \), if the time interval used for determination of the orbit is less (more) than 2 days.

6. The number of consequent unreliable updates to make the decision that a satellite is ”lost” – 2.
The simulated measurements on the break-up fragments were fed with time interval $dt = 20$ minutes. The overall number of measurements for 20 days for the first variant of the break-up was 10530, and for the second variant – 10115.

The quality of performance of the detection and tracking algorithm was evaluated using the following basic indicators:

1. The total number of measurements for all fragments $- n_{obs}$.
2. The number of detected and tracked satellites $- n_{det.SO}$.
3. The number of objects with alien measurements $- n_{SO.aliobs}$.
4. The number of alien measurements for tracked objects $- n_{al.obs}$.
5. The number of measurements correlated to tracked satellites $- n_{cor.obs}$.
6. The number of measurements not correlated with tracked satellites $- n_{unc.obs}$.
7. The number of lost objects $- n_{br.tr}$. 
Parameters for “significant” break-up as function of time
Parameters for “weak” break-up as function of time
Parameter Nunc.obz as function of time
Parameter Ndet.so as function of time
Parameter Nal.obz as function of time
Parameter \((\text{Nal.obz}/\text{Ncor.obz}) \times 100\%\) as function of time
AVERAGE CHARACTERISTICS of PRIMARY ORBIT DETERMINATION (DETECTION) for SATELLITES of DIFFERENT CLASSES for "SIGNIFICANT" ("WEAK") BREAK-UP

<table>
<thead>
<tr>
<th>sat.class</th>
<th>meas./day</th>
<th>meas.det</th>
<th>time det.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6 (6)</td>
<td>6 (6)</td>
<td>0.8 (0.8)</td>
</tr>
<tr>
<td>2</td>
<td>5 (5)</td>
<td>6 (7)</td>
<td>0.9 (1.0)</td>
</tr>
<tr>
<td>3</td>
<td>3 (3)</td>
<td>8 (10)</td>
<td>2.8 (3.5)</td>
</tr>
<tr>
<td>4</td>
<td>2 (2)</td>
<td>8 (10)</td>
<td>3.3 (4.3)</td>
</tr>
<tr>
<td>5</td>
<td>1 (1)</td>
<td>7 (10)</td>
<td>4.7 (8.1)</td>
</tr>
</tbody>
</table>
We can see the following from the figures.

1. For both variants of the break-up the detection and tracking algorithm has been fed by \( \approx 400 \) measurements. The number of uncorrelated observations \( n_{\text{unc.obs}} \) for significant (weak) break-up initially was increasing and by \( \approx 3.0 \ (\approx 3.7) \) days of simulation reached \( \approx 1400 \ (\approx 1770) \). Then the \( n_{\text{unc.obs}} \) decreased and stabilized at the level of 12 (36) by \( \approx 10 \ (\approx 20) \) days of simulation.

2. All 300 break-up fragments were detected and further stably tracked for the variants of ”significant” and ”weak” break-ups after 10.5 days and 18.5 days respectively.

3. The number of alien measurements for the objects initially increased and decreased further and with respect to the number of correlated measurements the decrease began from the level of 4%. The number of alien measurements for the objects of the ”weak” break-up is always by the order of magnitude greater than for the ”significant” break-up.
CONCLUSIONS

1. The considered detection and tracking algorithm demonstrated rather high resolution and efficiency of detection for the most simple case when the break-up occurs in the almost circular low Earth orbit with insignificant atmospheric drag. However, even for this case there are some unused possibilities for further enhancement of the efficiency. First we should mention:

a. using of single radar measurements for determination of orbits and testing their reliability;

b. using the fact that for the time of the break-up all the fragments were in the same point in space.

2. It is expedient to consider in future the more difficult case of the break-up in lower orbits where the atmospheric drag is the major perturbation factor. Here we face additional difficulties since the ballistic characteristics of the fragments (the functions characterizing for each fragment the change of area-to-mass ratio (in the direction of the velocity vector) in time) are not known. The simulation of measurements should take into account that this ratio is not uniform in time.

3. The equally difficult is the case of the break-up in the orbit with altitude more than 700 km, where solar radiation pressure exceeds the atmospheric perturbations. Here we have problems similar to the case of atmospheric drag - the area to mass ratio as function of the direction to the Sun is unknown. In addition for higher orbits regular acquisition of radar measurements is an issue, and without them we can hardly solve the detection and tracking problem.