LINEARIZED ORBIT COVARIANCE GENERATION AND PROPAGATION ANALYSIS VIA SIMPLE MONTE CARLO SIMULATIONS

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Introduction

- Realistic orbit error distributions can be used to drive a variety of space surveillance applications
  - Track association
  - Probability of collision calculations
  - Sensor tasking
- Many current approaches assume a Gaussian orbit error distribution represented by the covariance
  - Linearized dynamics used for propagation
- Are these approaches fundamentally flawed?
Small Step in Study of Covariance Realism

- Lots of reasons why covariance may not be realistic
  - Insufficiently modeled dynamics
    - Nonlinearities
    - Unmodeled accelerations
  - Poorly modeled observation errors
    - Non-Gaussian
    - Autocorrelated
- This research focused on the validity of linearized covariance propagation
  - Can we use \( P(t) = \phi P_0 \phi^T \) ?
Research Focused on Linearized Covariance Generation and Propagation

- Orbit error distribution should be Gaussian if
  - Obs errors are independent, zero mean, Gaussian
  - Dynamics are well represented
- This analysis enforces these assumptions in simulations to study impact of
  - Linearized dynamics
  - State representations
- What’s new?
  - Distinction of dynamic and coordinate linearization
  - Assessing practice of comparing covariance values to sample statistics (std deviations)
Simple Monte Carlo Simulations to Assess Covariance

- 1000 estimated orbits provides error distribution and k statistics

\[ k^2 = \delta X^T(t) P^{-1}(t) \delta X(t) \]

- Representative covariance results in k distribution that matches theoretical Gaussian distribution

- k should be constant in time under ideal conditions
Simulations Reflect LEO Space Surveillance Scenarios

- Single 2 minute radar pass scenario
  - Obs every 10 seconds, 30 m range noise, 36” az&el noise
  - ~100m of error at epoch and ~130km/day error growth
- Catalog-class scenario
  - Six passes of observation data to produce errors similar to observed catalog values
  - O[10m] of error at epoch and ~60m/day error growth

- 7000 km semimajor axis, near circular orbit
- Results generated in Cartesian, equinoctial, and curvilinear
  - DSST used for native equinoctial element formulation
- Perfect dynamic modeling, unbiased data with Gaussian noise
Sample $k$ distribution curves lie on top of each other
Catalog-Class Scenario: Cartesian Representation
Non-Gaussian after Few Days

- Is it nonlinear dynamics, linear comparison frame, or numerical issues?
Catalog-Class Scenario: Cartesian to Equincotial Element Transformation Matches Gaussian

- Used linear Jacobian transformation to convert Cartesian covariance to element space at comparison time
  - Still propagating in Cartesian space

- Linearization of Cartesian dynamics not the issue!
- Must be linear coordinate comparison frame issue
Catalog-Class Scenario: Cartesian Covariance Appears to Match Sample Statistics

- Poor measure?

- Same level of agreement in element case
Catalog-Class Scenario Summary

- Linearization of dynamics not a bad assumption
- Error distribution does not remain Gaussian in Cartesian reference frame
  - Transformation into element space mitigates issue
- Comparing covariance values to sample statistics (std deviations) does not appear to be a good measure
2-Minute Radar Pass: Cartesian Error Distribution
Non-Gaussian in Minutes!

- Covariance values still match sample statistics
2-Minute Radar Pass: Cartesian to Equincotial Element Transformation Much Improved

- Used linear Jacobian transformation to convert Cartesian covariance to element space at comparison time

- Nonlinearities drive non-Gaussian distribution after 1 day
- Numerical issues after Day 4
2-Minute Radar Pass: Equinoctial Element Error Distribution Non-Gaussian After 1 Day!

- Covariance values still match sample statistics through 10 days
- Condition number and inversion residuals look fine
- Better performance than Cartesian-to-Equinoctial after Day 3
  - Appears to be some advantage in element-based dynamics
Short Data Arcs Demonstrate Need for Nonlinearity Index or Nonlinear Techniques

• Equinoctial element results with various fit spans (10s/obs triplet)

  ![Graph](image)

  - Distribution remains mostly Gaussian and represented by covariance at epoch
  - Time it takes to go non-Gaussian sensitive to data spans and likely accuracy
Curvilinear coordinates are based on Cartesian but curve to follow the shape of the orbit.

Curvilinear \( k \) values are computed using 3x3 position + 3x3 velocity covariance.

- No position-velocity cross-correlation terms.
Two Minute Radar Pass Scenario Summary

- Nonlinear dynamics drive error distribution to be non-Gaussian in minutes to days
  - Using $P(t) = \phi P_0 \phi^T$ can be problematic
  - Recommend the use of nonlinear techniques or at least use of nonlinearity index prior to $P(t) = \phi P_0 \phi^T$

- Epoch error distributions appear to remain Gaussian and represented by covariance

- Setting some cross-correlation terms to zero when computing $k$ results in more consistent $k$ distributions
Conclusions

• Error distribution does not remain Gaussian in Cartesian reference frame

• For catalog-class errors, linearization of dynamics is not a bad assumption

• For short radar track, nonlinear dynamics drive error distribution to be non-Gaussian in minutes to days
  – Linearly propagated covariance matrix may not be sufficient to represent orbit error distribution

• Comparing covariance values to sample statistics (std deviations) does not appear to be a good measure
Catalog-Class Scenario: Numerical Stability Doesn’t Appear to be Primary Issue

- 64 bit Cartesian covariance inversion precision issue after day 4
  - k distribution moved away from Gaussian around day 3
- Element formulations are well behaved
Curvilinear Coordinates

- Created by Keric Hill, AIAA 2008-7211, AIAA/AAS Astrodynamics Specialist Conference, Honolulu, HI, Aug 2008
Elements with No Eccentricity-Mean Longitude Cross-Correlation Terms Very Consistent

- Propagating full element covariance
- Set $h$-$\lambda$ and $k$-$\lambda$ correlation terms to zero when computing $k$ values

Not useful for all applications but may be powerful approach for track association