Identification of maneuvers performed by low-thrust engines

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Identification and determination of parameters of the maneuvers
Identification and determination of the maneuvers can be divided into three stages (sub-problems)

• separation the intervals of passive motion within the interval of the measurements and determination of orbital parameters for these intervals;
• evaluation of the low thrust engine turn on and cut off times on the basis of orbital parameters determined at the previous step;
• updating of the turn on and cut off times and the generated acceleration using the available optical measurements.
Types of algorithms for determination of motion parameters

- Least squares method.
- Extended Kalman’s filter (EKF).
- Special methods.
Basic ideas of the special method (1)

• The estimate is determined from the condition of minimization of the functional depending both from the residuals of the measured values and their calculated analogs and on the values of the determined perturbations as well.

• The minimization of the functional is performed by iterations. For each step of the iteration process we determine the correction to the updated parameters.
Basic ideas of the special method (2)

• The correction is determined by the condition of the minimum functional for the linear system.

• The search for the minimum functional for the linear system leads to two sequences of recurrent formulas.

• The first sequence goes from the first measurement to the last one and provides determination of the correction for the time of the last measurement.

• The second sequence of the recurrent formulas goes from the last measurement to the first one and provides the recovery of the perturbations.
Model of spacecraft motion including perturbations

\[ \frac{dx}{dt} = F(t, x) + B(t)\xi(t) ; \quad x - \text{state vector}; \]

\[ F(t, x) - \text{vector-function}; \quad B(t) - \text{matrix, describing the noise}; \]

\[ \xi(t) - \text{white noise with intensity matrix } Q. \]

For obtaining the estimation the solution of the equation will be approximated by the following functions:

\[ x_A(t) = x_D(t) + x_P(t) ; \quad \frac{dx_D}{dt} = F(t, x_D) \]

\[ x_P(t) = \Phi(t, t_N)x_P(t_N) - \int_t^{t_N} \Phi(t, \tau)B(\tau)\xi(\tau)d\tau ; \quad \frac{d\Phi(t, t_N)}{dt} = \frac{\partial F}{\partial x}|_{x=x_D(t)} \Phi(t, t_N) \]

\[ \Phi(t_N, t_N) = E \]
Relationship between the state vectors for the times of neighboring measurements

\[
x_A(t_{i+1}) = x_D(t_{i+1}) + \Phi(t_{i+1}, t_i)(x_A(t_i) - x_D(t_i)) + v_i,
\]

\[
v_i = \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, \tau) B(\tau) \xi(\tau) \, d\tau,
\]

\[
Q_i = \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, \tau) B(\tau) Q(\tau) B^T(\tau) \Phi^T(t_{i+1}, \tau) \, d\tau,
\]

\[
x_A(t) = \begin{cases} 
  x_D(t_N) = x_N, & \text{if } t = t_N; \\
  x_D(t_i) + \Phi(t, t_i)(x_A(t_i) - x_D(t_i)) + v_{i-1}, & \text{if } t_i \leq t < t_{i+1}, 1 \leq i < N; \\
  x_D(t_0) + \Phi(t, t_0)(x_A(t_0) - x_D(t_0)), & \text{if } t_0 \leq t < t_1.
\end{cases}
\]
Measurements and estimation functional

Measurements:

\[(\Psi_i)_{\text{наб}} = \Psi_i(t_i, x(\cdot)) + \eta_i; \quad t_1, t_2, \ldots, t_N; \quad E[\eta\eta^T] = R_i.\]

A’priori data:

\[\bar{x}_0, \quad P_0\]

Estimation functional:

\[J = \frac{1}{2} \left( \left( x_A(t_0, q) - \bar{x}_0 \right)^T P_0^{-1} \left( x_A(t_0, q) - \bar{x}_0 \right) \right) + \]
\[+ \frac{1}{2} \sum_{i=1}^{N} \left( \left( \Psi_i \right)_{\text{наб}} - \Psi_i(t_i, x_A(t, q)) \right)^T R_i^{-1} \left( \left( \Psi_i \right)_{\text{наб}} - \Psi_i(t_i, x_A(t, q)) \right) + \]
\[+ \frac{1}{2} \sum_{i=0}^{N-1} v_i^T Q_i^{-1} v_i \quad .\]
Iterative minimization process

\[ J\left(\hat{x}_{0,N}, \hat{x}_{1,N}, \ldots, \hat{x}_{N,N}, \hat{v}_{0,N}, \ldots, \hat{v}_{N-1,N}\right) = \]

\[ = \frac{1}{2} \left(\hat{x}_{0,N} - \bar{x}_0\right)^T P_0^{-1} \left(\hat{x}_{0,N} - \bar{x}_0\right) + \]

\[ + \sum_{i=0}^{N-1} \frac{1}{2} \left[ (z_{i+1} - H_i^{(s)}\hat{x}_{i+1,N})^T R_{i+1}^{-1} \left( z_{i+1} - H_i^{(s)}\hat{x}_{i+1,N} \right) + \hat{v}_{i,N}^T Q_i^{-1} \hat{v}_{i,N} \right], \]

\[ \hat{x}_{i+1,N} = \Phi_i^{(s)} \hat{x}_{i,N} + \hat{v}_{i,N}, \quad i = 0, 1, \ldots, N - 1 \]

Matrices \( H_i^{(s)} \) and \( \Phi_i^{(s)} \) are calculated using the trajectory \( x_A^{(s-1)}(t) \),

obtained by previous iteration.
System of recurrent relationships.

Smoothing

\[
\hat{x}_{0,N} = P_0 \Phi_0^T \lambda_0 + \bar{x}_0, \quad \hat{x}_{i+1,N} = \Phi_i \hat{x}_{i,N} + Q_i \lambda_i, \quad \hat{v}_{i,N} = Q_i \lambda_i,
\]

\[
\lambda_i = \Phi_{i+1}^T \lambda_{i+1} + H_{i+1}^T R_{i+1}^{-1} \left[ z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right], \quad i = 0, \ldots, N - 1, \quad \lambda_N = 0
\]

\[
\hat{x}_{N,N} = \Phi_{N-1}^T \hat{x}_{N-1,N-1} + P_{N,N} H_N^T R_N^{-1} \left[ z_N - H_N \Phi_N \hat{x}_{N-1,N-1} \right]
\]

\[
\lambda_i = \Phi_{i+1}^T \lambda_{i+1} + H_{i+1}^T R_{i+1}^{-1} \left[ z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right], \quad \hat{x}_{i,N} = \Phi_{i+1}^{-1} \left[ \hat{x}_{i+1,N} - Q_i \lambda_i \right],
\]

\[
\hat{v}_{i,N} = Q_i \lambda_i, \quad \lambda_N = 0.
\]
Algorithm

• Determination of the correction to the state vector for the time of the last measurement and determination of the perturbation vectors by the results of the last iteration.

• Analysis of perturbation vectors. Calculation of RMS for the components of position and velocity. In case we see the perturbation vector for which the module of the components of position or velocity vectors exceeds 3 RMS, the interval of the measurements is divided into two.

• Go to processing of the first interval.
Algorithm’s performance (example)
## Determined times of perturbations

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Relative residuals as function of time

Relative residuals

Days of flight
Results of impulse determination.

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<th>wMRSE</th>
<th>mDV, м/с</th>
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# Results of impulse determination.

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Conclusions

• The method and the algorithm for determination of bends of the trajectories due to the maneuvers performed by low thrust engines;

• The developed algorithm is in the phase of trial operation processing optical measurements acquired by ISON.