Identification of Maneuvers Executed by Low-Thrust Engines

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In this paper problems of identification and determination of parameters of maneuvers executed by low-thrust engines using optical measurements are considered. The complexity of this problem is determined by the long period of the engines work and the small accelerations produced by the engines. The problem of identification and determination of maneuvers executed by low-thrust engines can be divided into three stages: division of the observation interval into sections of “passive” motion and determination of the orbital parameters in these sections; estimation of intervals of the low-thrust engine firing and cutoff by the orbital parameters determined in the previous stage; correction of the engine firing and cutoff intervals and the acceleration produced by them using the available optical measurements. In this report the problem of division of the observational interval into the sections of the space vehicle (SV) “passive” motion and determination of the motion parameters in these sections is considered. The proposed algorithm is used in processing of the measurements of Scientific Network of Optical Instruments for Astrometric and Photometric Observations (NSOI AFN³).

Identification and determination of parameters of maneuvers executed by low-thrust engines using optical measurements is a challenging problem. Its complexity is determined by the long period of the engines work and the small accelerations produced by them. For example, a SV, executing in the geostationary orbit (GEO) a maneuver for a change of the stand point, has to obtain a characteristic speed of about 2-3 m/s. This characteristic speed is reached as the result of the low-thrust engine work within a day, or several days. To keep the SV in the GEO in the neighborhood of the determined stand point, a correction is carried out with the consumption of the characteristic speed of about 0.1-0.2 m/s. To carry out such a correction the low-thrust engine is fired for an hour or several hours.

The problem of identification and determination of maneuvers executed by low-thrust engines can be divided into three stages (subproblems):
- Division of the observational interval into sections of “passive” motion and the determination of the orbital parameters in these sections;
- Estimation of the intervals of the low-thrust engine firing and the cutoff by the orbital parameters determined in the previous stage;
- Correction of the engine firing and cutoff intervals, and the acceleration produced by them, using the available optical measurements.

The problem of estimation of the low-thrust engines firing and the cutoff intervals by the available orbital parameters is close to the optimal control problems in its definition. That is why for its solution it is expedient to use the experience of solution of problems of optimal impulse maneuvering on near-circular orbits, accumulated in the period of more than 30 years at the Institute of Applied Mathematics of Russian Academy of Sciences. In the Institute of Applied Mathematics of Russian Academy of Sciences a theory has been developed for such maneuvering. The effective methods have been proposed for the solution of all major problems met in practical work [1-8]. These methods have been used in the calculation of maneuvering parameters of

³ International Scientific Optical Network (ISON)
“Soyuz” and “Progress” SV, orbital modules “Kvant”, “Spektr”, “Zvezda”, etc. Because of their simplicity and reliability, these methods are widely used, for example, a similar algorithm was used in the calculation of ATV maneuvers [9].

The well-proven impulse solutions form the basis of a calculation method for low-thrust maneuvers [10]. The impulse solutions make it possible to find the midpoints of active sections and the thrust vector orientation in the case of a three-dimensional orbit transfer. Then the maneuver’s time is determined analytically. By means of the inner iterative procedure all the maneuver’s parameters are adjusted in order to form all the final orbit elements with the necessary precision. The outer iterative procedure makes it possible to take into account the effect of the eccentricity of the gravity field, atmosphere, gravitational attraction of Moon, Sun, etc.

The transfers are considered with one, two, or more maneuvers, as, for example, $\Delta V$ of a transfer with three maneuvers can be three times less than $\Delta V$ of a transfer with two maneuvers.

When solving the problems, it is assumed that the thrust is close to constant in the course of the maneuver’s execution and the thrust vector orientation is fixed in the orbital or inertial reference system. In some cases it is optimal to execute one of the maneuvers with the thrust vector orientation fixed in the inertial reference system, and in other cases — in the orbital reference system.

As optimization is performed in the space of one or two variables, the problem is solved almost immediately. It enables to use the program data in an operational simultaneous analysis of the motion of numerous SV.

Another method of estimation of low-thrust engine firing and cutoff intervals by the available orbital parameters has been proposed in [11].

In this report a problem of division of the observational interval into the sections of the SV “passive” motion and the determination of the motion parameters in these sections is considered. Presently two types of algorithms are used for the determination of SV motion parameters:

- Least-Squares Method;
- Extended Kalman Filter (EKF).

The long-term practice has shown that the least-squares method is a very reliable method for determination of parameters. Estimation is obtained as the result of a functional minimum search, which is a sum of weighted discrepancies between the measured values and their calculated analogs. In this case the calculated analogs are functionally dependant on the adjusted parameters. The feature of this method is that a SV motion model has to be quite accurate. It is not allowed to have big disturbances, which can not be defined in the form of dependencies on the adjusted parameters. That is why the application of the least-squares method in the case of unknown low-thrust operation intervals causes certain problems. The least-squares method can be successfully used at a stage when there is already an estimation of the engine firing and cutoff moments and the acceleration produced by them.

Extended Kalman Filter is an empirical extension of the Kalman Filter for linear systems to a nonlinear filter. Extended Kalman Filter assumes the presence of an unknown noise having an effect on the system. The noise characteristics are determined by its covariance matrix. The feature of this method is that the current values of the estimated parameters have to be quite close to their actual values. Besides, the method requires having no long periods of time without measurements. The above circumstances prevent using the Extended Kalman Filter for the solution of a problem of division of the observational interval into the sections of “passive” motion and the determination of the orbital parameters in these sections.

In this report a method proposed in [12] is offered for the solution of the problem of division of the observational interval into the sections of “passive” motion and determination of the orbital parameters in these sections. The substance of the method is that the estimation is determined on conditions of minimization of a functional, dependent both on the discrepancies of the measured
values and their calculated analogs, and on the values of the determined disturbances. The functional is minimized iteratively. At each stage of the iteration process a correction is determined to the desired parameters. The correction is determined on conditions of the functional minimum for a linear system. The search for the functional minimum for a linear system results in two sequences of recurrent formulas. The first sequence of the recurrent formulas goes from the first measurement to the last measurement and makes it possible to determine the correction in the moment of the last measurement. The second sequence of the recurrent formulas goes from the last measurement to the first measurement and makes it possible to recover the disturbances. The first sequence of the recurrent formulas is equivalent to the recurrent formulas of the Kalman Filter for a linear system. The second sequence of the recurrent formulas is called smoothing [13].

It is necessary to note a well-known fact that, in absence of disturbances, the correction calculated under the recurrent formulas of the Kalman Filter agrees with the correction calculated from the solution of a system of normal equations of the least-squares method.

The main idea of the proposed algorithm is in the usage of the smoothing results for the determination of possible disturbance moments.

It is proposed to execute the following algorithm. Let the SV motion be described with the following differential equation:

\[
\frac{dx}{dt} = F(t, x) + B(t)\xi(t),
\]

(1)

where 
- \(x\) — the state vector;
- \(F(t, x)\) — the vector function;
- \(B(t)\) — the matrix describing the impact of noise on the system;
- \(\xi(t)\) — the white noise with zero mathematical expectation and determined intensity matrix \(Q(t)\) of order 3×3.

The initial conditions for the system (1) are established with the a priori vector \(\bar{x}_0\) and its covariance matrix \(P_0\).

In the moments of time \(t_1, t_2, \ldots, t_N\) there are changes of functions \(\Psi_1, \Psi_2, \ldots, \Psi_N\). Let’s denote the measured value of function \(\Psi_i\) by \((\Psi_i)_{t_{i}}\). For each moment of time \(t_i\) it is true that

\[
(\Psi_i)_{t_{i}} = \Psi_i(t_i, x(t_i)) + \eta_i,
\]

(2)

where \(\eta_i\) is a random vector with zero mathematical expectation and covariance matrix \(R_i\).

In the linear case, (2) has the following form

\[
z_i = H_i(t_i)x(t_i) + \eta_i
\]

(2')

where \(z_i\) is the vector of dimension parameters \(r_i\) measured in the moment of time \(t_i\), \(H_i(t_i)\) is the dimension matrix \(r_i \times n\).

Representation as a parameter \(x(\cdot)\) of function \(\Psi_i\) means that the function \(\Psi_i\) depends not on the momentary value of the state vector, but on the function \(x(t)\) which is the solution of equation (1).

Let’s approximate the solution of equation (1) in the interval \([t_0, t_N]\) with the following types of functions:
\[ x_A(t) = x_D(t) + x_P(t), \]

where

\[ \frac{dx}{dt} = F(t, x_D). \] (3)

\[ x_P(t) = \Phi(t, t_N) x_P(t_N) - \int_t^{t_N} \Phi(t, \tau) B(\tau) \xi(\tau) d\tau. \] (4)

The matrix function \( \Phi(t, \tau) \) satisfies the equation

\[ \frac{d\Phi(t, t_N)}{dt} = \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} \Phi(t, t_N) \] (5)

on the initial conditions \( \Phi(t_N, t_N) = E \).

\( x_P(t_N) \) is the vector parametrizing the family of functions \( x_P(t) \).

Let’s show that \( x_A(t) \) actually approximates the solutions of system (1). Indeed,

\[ \frac{dx}{dt} = F(t, x_D) + \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} x_P(t) + B(t) \xi(t) - \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} x_P(t) + B(t) \xi(t) d\tau. \]

Grouping the terms containing \( \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} \) and using the relation (4), we have:

\[ \frac{dx}{dt} = F(t, x_D) + \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} x_P(t) + B(t) \xi(t). \]

As \( F(t, x) = F(t, x_D + x_P) \approx F(t, x_D) + \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} x_P(t) \) than \( x_A(t) \) approximates the solution of equation (1).

Let’s note that \( x_P(t) \) satisfies the linear stochastic differential equation:

\[ \frac{dx}{dt} = \frac{\partial F}{\partial x} \bigg|_{x=x_P(t)} x_P(\xi(t)) + B(t) \xi(t). \] (6)

The dependence between the state vectors \( x_A(t_i) \) in the discrete moments of time \( t_0, t_1, ..., t_N \) can be expressed with the difference equation:

\[ x_A(t_{i+1}) = x_D(t_{i+1}) + \Phi(t_{i+1}, t_i) \left( x_A(t_i) - x_D(t_i) \right) + \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, \tau) B(\tau) \xi(\tau) d\tau. \] (7)

Let’s denote the random vector \( \int_{t_i}^{t_{i+1}} \Phi(t_{i+1}, \tau) B(\tau) \xi(\tau) d\tau \) by \( v_i \). This random vector has zero mathematical expectation and covariance matrix...
Let’s assume that \( x_p(t_N) = 0 \). Then the initial conditions vector \( x_D(t_N) = x_A(t_N) = x(t_N) \) expressly determines the values of the function vector \( x_A(t) \) in the discrete points: \( t_0, t_1, \ldots, t_N \). Although, when calculating the values of the function \( \Psi_i(t, x_A(t)) \), it is necessary to know the dependence \( x_A(t) \) in the neighborhood of each moment of time \( t_i \). Let’s present this dependence in the following form:

\[
\begin{align*}
x_A(t) = \begin{cases}
x_D(t_N) = x_N, & \text{if } t = t_N; \\
x_D(t_i) + \Phi(t_i, t_i)[x_A(t_i) - x_D(t_i)] + v_{i-1}, & \text{if } t_i < t < t_{i+1}, 1 \leq i < N; \\
x_D(t_0) + \Phi(t_0, t_i)[x_A(t_0) - x_D(t_0)], & t_0 < t < t_i.
\end{cases}
\end{align*}
\] (9)

Therefore, a parametric dependence \( x_A(t, q) \) is built, where \( q \) is the vector of the determined parameters consisting of the components of vectors \( x(t_N), v_0, \ldots, v_{N-1} \).

As well as in the linear case, the estimation quality criterion is a functional containing the squared weighted deviation of the a priori specified state vector from its calculated value, and also, the squared discrepancies of measurements and weighted cumulative disturbances between the measurements. This functional can be represented in the following form:

\[
\begin{align*}
J & = \frac{1}{2} \left[ (x_A(t_0, q) - \bar{x}_0)^T P_0^{-1} (x_A(t_0, q) - \bar{x}_0) \right] + \\
& + \frac{1}{2} \sum_{i=0}^{N-1} \left( (\Psi_i)_{\bar{\alpha} \bar{\alpha}} - \Psi_i(t, x_A(t, q)) \right)^T R_i^{-1} \left( (\Psi_i)_{\bar{\alpha} \bar{\alpha}} - \Psi_i(t, x_A(t, q)) \right) + \\
& + \frac{1}{2} \sum_{i=0}^{N-1} v_i^T Q_i^{-1} v_i.
\end{align*}
\] (10)

If the functional minimum (10) is searched for by the Newton method, the corrections of each iterative stage minimize the quadratic form obtained from (10) by means of substitution of nonlinear dependences for the linear members of the Taylor series. It means that on the iteration stage \( s \) a problem of optimal estimation of the linear system (1, 2') state. The matrixes \( A^{(s)}(t) \) and \( H^{(s)}_i(t), i = 1, \ldots, N \) of that system are calculated by the following formulas:

\[
A^{(s)}(t) = \frac{\partial F}{\partial x} \bigg|_{x = x_A(t, q^{(s)})},
\] (11)

\[
H^{(s)}_i(t) = \frac{\partial \Psi_i}{\partial x} \bigg|_{x = x_A(t, q^{(s)})}.
\] (12)
The values of variables for which the functional maximum is reached

\[ J(\hat{x}_{0,N}, \hat{x}_{1,N}, \ldots, \hat{x}_{N,N}, \hat{v}_{0,N}, \ldots, \hat{v}_{N-1,N}) = \]

\[ = \frac{1}{2} (\hat{x}_{0,N} - \bar{x}_0)^T P_0^{-1} (\hat{x}_{0,N} - \bar{x}_0) + \]

\[ + \sum_{i=0}^{N-1} \frac{1}{2} \left( z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right)^T R_{i+1}^{-1} \left( z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right) \]

\[ + \psi_{i,N}^T Q_i^{-1} \hat{v}_{i,N} \quad (13) \]

with the limitations:

\[ \hat{x}_{i+1,N} = \Phi_i^{(s)} \hat{x}_{i,N} + \hat{v}_{i,N}, \quad i = 0, 1, \ldots, N-1 \quad (14) \]

satisfy the system of equations:

\[ \hat{x}_{0,N} = P_0 \Phi_0^T \lambda_0 + \bar{x}_0, \]

\[ \hat{x}_{i+1,N} = \Phi_i \hat{x}_{i,N} + Q_i \lambda_i, \]

\[ \hat{v}_{i,N} = Q_i \lambda_i, \quad (15) \]

\[ \lambda_i = \Phi_i^{(s)} \lambda_{i+1} + H_{i+1}^{(s)} R_{i+1}^{-1} \left[ z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right], \quad \text{for} \ i = 0, \ldots, N-1, \]

\[ \lambda_N = 0. \]

This system of equations falls into two systems of recurrent relations. One system connects the a priori estimation to the final correction at the moment of the last measurement. A forward motion on measurements takes place. This recurrent system agrees with the Kalman Filter algorithm:

\[ \hat{x}_{N,N} = \Phi_{N-1}^{(s)} \hat{x}_{N-1,N-1} + P_{N,N} H_N^{(s)} R_N^{-1} \left[ z_N - H_N \Phi_N \hat{x}_{N-1,N-1} \right]. \quad (16) \]

The other recurrent system goes from the last measurement to the first one and performs smoothing. This system makes it possible to determine the estimates of the state vectors (corrections) in the moment of time \( t_i \) by the information in the interval from \( t_1 \) to \( t_N \), and also, recover the value of the noise vectors:

\[ \lambda_i = \Phi_i^{(s)} \lambda_{i+1} + H_{i+1}^{(s)} R_{i+1}^{-1} \left[ z_{i+1} - H_{i+1} \hat{x}_{i+1,N} \right], \]

\[ \hat{x}_{i,N} = \Phi_i^{-1} \left[ \hat{x}_{i+1,N} - Q_i \lambda_i \right], \]

\[ \hat{v}_{i,N} = Q_i \lambda_i, \]

\[ \lambda_N = 0. \]

Let’s consider the algorithm application by the example of processing optical measurements obtained for the space object (SO) 2000-031A (Express 3A). Scientific Network of Optical Instruments for Astrometric and Photometric Observations (NSOI AFN)\(^1\) carried out optical observation of the above SO in the interval from June 18, 2009 to September 29, 2009. In the above interval of time SO 2000-031A executed a number of maneuvers, with the result that at first a transfer from the point with the longitude of 349° to the stand point with the longitude 38° was made, and then the SO was transferred to the orbit with a height of more than 400 km in relation to

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\(^1\) The Network NSOI AFN is coordinated by M.V. Keldysh Institute of Applied Mathematics
the geostationary orbit height. The ground track of the SO with the international number 2000-031A is shown in the Figure 1 and the dependence of the subsatellite point on time is shown in Figure 2.

In the result of application of the above algorithm, the following maneuver execution moments were detected under the UTC scale:

<table>
<thead>
<tr>
<th>Date and time, UTC</th>
<th>wMRSE</th>
<th>mDV, m/s</th>
<th>vR, m/s</th>
<th>vN, m/s</th>
<th>vB, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009/06/19</td>
<td>1.80</td>
<td>1.280</td>
<td>-0.647</td>
<td>-1.069</td>
<td>-0.279</td>
</tr>
<tr>
<td>2009/08/04</td>
<td>5.26</td>
<td>10.915</td>
<td>10.373</td>
<td>3.317</td>
<td>-0.719</td>
</tr>
<tr>
<td>2009/08/15</td>
<td>0.25</td>
<td>0.154</td>
<td>0.154</td>
<td>-0.001</td>
<td>0.005</td>
</tr>
<tr>
<td>2009/08/18</td>
<td>0.29</td>
<td>0.019</td>
<td>0.019</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>2009/08/20</td>
<td>0.30</td>
<td>1.812</td>
<td>-0.358</td>
<td>1.715</td>
<td>-0.462</td>
</tr>
<tr>
<td>2009/08/22</td>
<td>0.26</td>
<td>1.579</td>
<td>0.487</td>
<td>1.484</td>
<td>-0.231</td>
</tr>
</tbody>
</table>

Let’s consider the relation of the value of discrepancy between the measured and the calculated values to the a priori mean-square deviation (MSD) of the measurement error. This relation is a dimensionless value and shall not be more than 3 with the measurement errors corresponding to the priori values of the errors MSD and the right motion models. Let’s call this relation a relative discrepancy. The graphs of dependency on time of relative discrepancies of right accession and declination are shown in Figure 3.

Further the impulse execution moments and the impulse values were adjusted by the corresponding measurement intervals. In this case the state vector in the beginning of the measurement interval was simultaneously determined, as well as the maneuver time and the impulse value. The results are represented in the following Table 1. In that table column wMRSE contains MSD of the relative discrepancies, column mDV contains the adjusted value of the impulse module. Columns vR and vN contain the projections of the adjusted impulse vector on the radial and transversal directions, correspondingly. Column vB contains the projections of the adjusted impulse vector on the direction orthogonal to the orbital plane.

Table 1 contains the parameters making it possible to estimate the algorithm application reliability. The impulse estimation at 2009/08/04 5:01:38 UTC can not be called satisfactory as MSD of the relative discrepancies is more than 3 and the determined impulse has a big radial component. That is why this maneuver can only be spoken about as a detected maneuver. The estimation of the maneuver parameters is unreliable. Maneuver 2009/08/18 0:19:18 has a low characteristic speed. Probably, this maneuver didn’t take place. Maneuver 2009/08/27 3:32:36 has a big radial component which is indicative of an incorrect determination of the maneuver time.

Consequently, in the existing state among 10 detected maneuver facts, only 9 were reliable. Among 9 estimations only 7 were reliable.

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Figure 1. The ground track of a SO with the international number 2000-031A in the interval of time from June 18, 2009 to September 29, 2009

Figure 2. Dependency on the longitude time of the subsatellite point of a SO with the international number 2000-031A in the interval of time from June 18, 2009 to September 29, 2009. On the axis
of abscissas the time in days is specified from 0 hours of June 18, 2009. On the axis of ordinates the longitude of the subsatellite point is specified.

Figure 3. Graph of dependency on time of relative discrepancies of direct accession (the black line) and declination (the blue line). On the axis of abscissas the time in days is specified. On the axis of ordinates the relative discrepancy is specified.

References

12. Tuchin A.G., Determination of a SV Motion Parameters by the Results of Measurements with the Presence of Noise in the Dynamic System, Preprint by M.V. Keldysh Institute of Applied Mathematics of Russian Academy of Sciences # 2, 2004