Guaranteed Approach For Orbit Determination With Limited Error Measurements

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THE PREVENTING POTENTIAL COLLISIONS BETWEEN ARTIFICIAL SPACE OBJECTS (SO) IS MAIN PURPOSE

Two types of the errors are possible: the miss of collision (probability $\alpha$) and a false alarm (probability $\beta$).

The errors $\alpha$ and $\beta$ depend on:

1. the accuracy of the method used for determination of SO predicted position.
2. correspondence level of actual object predicted position errors to their calculated values.

IS IT POSSIBLE TO REDUCE THE FREQUENCY OF THE MISS OF COLLISION AND THE FREQUENCY OF THE FALSE ALARM ESSENTIALLY USING ALREADY AVAILABLE MEASURING INFORMATION?
FEATURES OF EXISTING ALGORITHMS

1. The statistical approach is applied.

2. The algorithms are based on a method of the least squares and its recurrent modifications.

3. The algorithms have the property of a global optimality for Gaussian distribution of measurement errors and the property of an optimality in a class of linear algorithms for any error distributions.

4. The errors of real measurements are non-Gaussian. The nonlinear algorithms may exist too. Therefore the existing algorithms does not provide the minimal errors of SO predicted positions generally.

5. The algorithms does not provide the computation of correlation matrixes of object predicted position errors, which correspond to actual values of these errors.
IS IT POSSIBLE TO CREATE THE ALGORITHMS, WHICH CALCULATE SPACE OBJECT PREDICTED POSITIONS USING THE MEASUREMENTS AND WHICH HAVE THE FEATURES LISTED BELOW?

1. CORRESPONDENCE TO THE PROPERTIES OF REAL ERRORS (the distribution is not known, the non-abnormal values are limited by known constants) IS BETTER, THAN IN THE EXISTING ALGORITHMS.

2. CALCULATED VALUES OF THE ERRORS CORRESPOND TO THEIR REAL VALUES.

3. ACTUAL ERRORS OF SPACE OBJECT PREDICTED POSITION DEFINITION ARE NOT GREATER, THAN THE ERRORS OF THE LEAST SQUARES METHOD

1. YES 2. YES 3. sometimes YES
STATEMENT OF THE PROBLEM

Let \( y = (y_1, y_2, \ldots, y_n) \) are the measurements of some functions \( I(x) = (i_1(x), i_2(x), \ldots, i_n(x)) \), where \( x \) is of \( m \)-dimensional vector of parameters. Let \( t_1 \leq t_2 \leq \ldots \leq t_n \) are times of measurements. Let \( \mathbf{x} = (x_1, x_2, \ldots, x_n) \) are measurement errors limited by the known constants \( \mathbf{x} \) above, i.e.,

\[
y = I(x) + \delta \quad |\delta_k| \leq \varepsilon \quad k = 1, 2, \ldots, n
\]

Let target function is \( z = S(x) \).

In our problem \( x \) are orbit parameters and \( z \) is position vector in a certain time \( t \).

It is required to find an estimation \( z_{ga} = z_{ga}(y) \) of parameter \( z \) with the least errors \( z_{ga} - z \) and to estimate maximal values of these errors \( \max |z_{ga} - z| \).

LINEARIZATION

LS estimation: \( x_{LS} = \arg \min_x (y - I(x))' \cdot W \cdot (y - I(x)), \quad W \)– weight matrix of measurements.

Linearised problem: \( y = I \cdot x + \delta \quad z = S \cdot x \quad |\delta_k| \leq \mathbf{x} \quad k = 1, 2, \ldots, n \)

\( y := y - I(x_{LS}) \quad x := x - x_{LS} \quad z := z - z_{LS} \quad I = \partial I/\partial x(x_{LS}) \quad S = \partial S/\partial x(x_{LS}) \)
Not statistical approach

1. In 1960-1970 in mathematics the theory of optimum algorithms in the normalized spaces has been developed. It was applied to the decision of mathematical problems of approximation in the theory of functions, the differential both integrated equations and other abstract mathematical disciplines.

2. In 1980th years this theory has been adapted to the decision of estimating and predicting problems at the only thing of assumptions of measurement errors - limitation by known values. Since 1970-1980 years this approach has found practical application in applied areas: power, electronics, chemistry, biology, medicine, etc.

3. In space information-analytical systems the USA and Russia the given approach was not applied. The possible reasons are complexity, uncertainty in its efficiency, unwillingness to reconstruct already created systems radically.

THE WORK IS DEVOTED TO THE DEVELOPMENT AND THE RESEARCH OF NEW METHODS BASED ON THIS APPROACH FOR ORBITS DETERMINATION.
STATEMENT OF THE PROBLEM FOR NON-STATISTICAL APPROACH

\[ \mathbb{R} = \{ r \} \]  — the linear normalized space

Norm  \( n(r) = \| r \| \)  — a function on  \( \mathbb{R} \), which satisfies to the conditions:

- nonnegative  \( n(r) \geq 0 \) (if  \( n(r) = 0 \), then  \( r = 0 \))
- linearly-scalable  \( n(\alpha \cdot r) = |\alpha| \cdot n(r) \)
- convex  \( n(r_1 + r_2) \leq n(r_1) + n(r_2) \).

Example of norm:  \( l_p = (\sum_k |r_k|^p)^{1/p}, \)  \( r_k \)  \( - k \)-th component of vector  \( r \).

-  \( p = 2 \)  regular vector magnitude  \( l_2 = (\sum_k |r_k|^2)^{1/2} \),
-  \( p = 1 \)  the sum of modules of vector components  \( l_1 = \sum_k |r_k| \),
-  \( p = \infty \)  the maximum modulus of component  \( l_\infty = \max_k |r_k| \).

\( X \)  — unknown element space (\( m \)-dimensional)
\( Y \)  — known measurement space (\( n \)-dimensional,  \( n \geq m \)).
\( Z \)  — solution space (\( p \)-dimensional).

In our applied problem the elements of space  \( X \)  are orbit parameters,  \( Y \)  are the measurements of known functions of these parameters,  \( Z \)  are position coordinates in a certain time.

The condition of limitation of measurement errors  \( |\delta_k| \leq \varepsilon \)  \( k = 1,2,\ldots,n \)  is equivalent to a condition  \( \| \delta \| \leq \varepsilon \), where  \( l_\infty \)  is understood as norm.
S: $X \rightarrow Z$ solution operator $S(x)$.

I: $X \rightarrow Y$ information operator $I(x)$ (mapping $I(x)$ is to be one-to-one).

The measurement $y \in Y$ is known. It is necessary to find a solution element $z \in Z$.

If $y = I(x)$ (measurement is correct), then $x = I^{-1}(y)$ and $z = S(x)$ The problem is solved.

Usually $y \neq I(x)$ and $y - I(x) = \epsilon$ (measurement error) $\|\epsilon\| \leq \epsilon$ ($\epsilon$ — known constant).

A: $Y \rightarrow Z$ algorithm (gives the approximation of solution element $z$ by the $y$ measurement.

$U_x(y) = \{x \in X \mid I(x) - y \leq \epsilon\}$ — the uncertainty area $x$ for the measurement $y$

$U_y(x) = \{y \in Y \mid I(x) - y \leq \epsilon\}$ — the uncertainty area $y$ for unknown element $x$

$U_z(y) = S\{U_x(y)\}$ — the uncertainty area $z$ for the measurement $y$
\(X\) — unknown element space \((m\text{-dimensional})\)
\(Y\) — known measurement space \((n\text{-dimensional}, \ n \geq m. )\)
\(Z\) — solution space \((p\text{-dimensional})\).
THE EXAMPLES OF THE ALGORITHMS

1. Correct algorithm \( A_{cr} \), \( A_{cr}(I(x)) = S(x) \) for each \( x \in U_x(y) \)
   
   It is similar to the concept of unbiased parameter estimate in statistical approach.

2. Interpolated algorithm \( A_{in} \), \( A_{in}(y) \supseteq U_z(y) \) for each \( y \in U_y(x) \)
   
   \( D = \max_{(z_1, z_2 \in U_x(y))} \| z_1 - z_2 \| \) — the diameter of uncertainty area \( U_z(y) \) is the guaranteed ranges of any interpolated estimate parameters for any realization of measurement errors. It is serves as a one-dimensional analog of Fisher information matrix in statistical approach. The interpolated algorithm has property of adaptation to real measurement errors. The more frequent error values close to the maximum possible value and having different signs are in particular realizations of measurements, the less is such algorithm error.

3. Central algorithm \( A_{cn} \), \( A_{cn}(y) = \arg \min_z (\max_{v \in U_z(y)} \| z - v \|) \)
   
   \( A_{cn}(y) \) — Chebyshev center of uncertainty area \( U_z(y) \), i.e. a point of set \( Z \), for which the maximum distance to points \( U_z(y) \) is minimal.

4. Projected algorithm \( A_{pr} \), \( A_{pr}(y) = S(x_{pr}) \)
   
   \( x_{pr} = \arg \min \| y - I(x) \| = \min_{x} U_x(y, \varepsilon) \)

   \( A_{pr} \) is the projection of \( Y \) space \( n \)-vector to \( m \)-dimensional subspace \( I(X) \). \( A_{pr} \) is a limit of uncertainty area \( U_x(y) = U_x(y, \varepsilon) \), when it is subtended to the point by decreasing of upper limit of errors \( \varepsilon \). Projective estimation \( x_{pr} \) is interpolated and robust (it is independent from \( \varepsilon \)).
ONE DIMENTIONAL CASE (m=1)

\[ y_k = x + \delta_k \quad |\delta_k| \leq \varepsilon \quad (k=1,2,\ldots,n) \quad z = x \quad U_x(y) = \{ x \mid R_1 \; y_{\text{max}} - \varepsilon \leq x \leq y_{\text{min}} + \varepsilon \} \]

\[ x_{ga} = A_{cn}(y) = A_{pr}(y) = (y_{\text{max}} + y_{\text{min}})/2 \]

\[ D = 2\varepsilon - y_{\text{max}} + y_{\text{min}} \quad |x_{ga} - x| \leq \varepsilon - (y_{\text{max}} - y_{\text{min}})/2 \]
Density of uniform, triangle and Gauss-3σ distribution in [ε, -ε] interval
The comparison of RMS estimation errors LS and GA (asymptotic)

<table>
<thead>
<tr>
<th>estimation formula</th>
<th>RMS estimation errors for different distribution errors</th>
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<tbody>
<tr>
<td></td>
<td>uniform</td>
</tr>
<tr>
<td>$x_{HK} = (\Sigma y_i)/n$</td>
<td>$\sigma$/sqrt(n)</td>
</tr>
<tr>
<td>$x_{\Pi} = (y_{\text{max}} + y_{\text{min}})/2$</td>
<td>$\approx 2.5\cdot\sigma/n$</td>
</tr>
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</table>

The ratio of mean errors of GA and LS estimations (simulation –1000)

<table>
<thead>
<tr>
<th>error measurements distribution</th>
<th>number of measurements $n$</th>
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<tr>
<td></td>
<td>10</td>
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<tr>
<td>uniform</td>
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</tr>
<tr>
<td>triangular</td>
<td>1.10</td>
</tr>
<tr>
<td>gaussian-3$\sigma$</td>
<td>1.20</td>
</tr>
</tbody>
</table>
THE ALGORITHM ERRORS AND OPTIMALITY

\[ e(A, x, y) = \| A(y) - S(x) \| \] — the error of algorithm \( A \)

\[ e_x(A) = \max_{y \in U_y} e(A, x, y) \] — the error of algorithm \( A \) for the worst measurement errors

\[ e_y(A) = \max_{x \in U_x} e(A, x, y) \] — the error of algorithm \( A \) for the worst \( x \in U_x \) (y)

**optimal algorithm** \( A_{opt,y} \)

\[ e_x(A_{opt,y}) = \min_A e_x(A) \] — the algorithm minimizes the error \( e_x(A) \) for each of \( A \) and \( x \)

**optimal algorithm** \( A_{opt,x} \)

\[ e_y(A_{opt,x}) = \min_A e_y(A) \] — the algorithm minimizes the error \( e_y(A) \) for each of \( A \) and \( y \)

The central algorithm is \( A_{opt,x} \) algorithm.

**optimal algorithm** \( A_{opt} \)

\[ e(A_{opt}) = \min_A e_x(A) = \min_A e_y(A) \] — the algorithm is optimal for each of the two types
THE ALGORITHMS IN NORM $l_\infty$ AND THEIR OPTIMALITY

The problem is linear: $I(x) = I \cdot x$  $S(x) = S \cdot x$  $I$, $S$ — matrixes $n \times m$ and $p \times m$.

1. $A_{cn} = A_{opt}$ — the central algorithm is optimal.

   $z_{cn} = A_{cn}(y) = 0.5 \cdot (z_M + z_m)$  $z \leq \min \{ s' \cdot x \} \leq \max \{ s' \cdot x \} \leq y - i' \cdot x \leq \max \{ s' \cdot x \}$  $k=1,2,\ldots,p$  $q=1,2,\ldots,n$

   $z_{k,m}, z_{k,M}$ — components $p$-vectors $z_m, z_M$  $s', i'$ — $k$-th and $q$-th rows of matrixes $S, I$

   It is equivalent to $2p$ linear programming problems in $m$-dimensional space $X$.

2. The projective algorithm $A_{pr}$ is interpolated and it is not optimal.

   The projective estimation $x_{pr}$ is a solution of linear programming problem:

   $\min \alpha_{x:y} \leq y - i' \cdot x \leq \alpha_{y:x} \ U_{x,y} = \{ x:y \} X: \parallel I \cdot x - y \parallel \leq \alpha \cdot \varepsilon \} \ q=1,2,\ldots,n$  

   in $(m+1)$-dimensional space $(X, y)$.

   $x_{pr}$ loses in error $e_y$ to the optimal estimation $x_{cn}$ not more than two times.

THE CENTRAL AND PROJECTIVE ALGORITHMS ARE NONLINEAR
THE ALGORITHMS IN NORM $l_2$ AND THEIR OPTIMALITY

The problem is linear: $I(x) = I \cdot x$  $S(y) = S \cdot y$  $I, S$ — matrixes $n \times m$ and $p \times n$.

$A_{cn} = A_{pr} = A_{opt}$ — central, projective and optimal algorithms are the same.

An estimate of the solution parameters is the estimate of LS $z_{cn} = z_{pr} = z_{LS} = S \cdot (I' \cdot I)^{-1} \cdot I' \cdot y$.

The estimate $z_{LS}$ is linear function of measurements $y$.

The estimation $z_{LS}$ is neither optimal nor interpolated in norm $l_\infty$.

THE ALGORITHMS IN NORM $l_1$ AND THEIR OPTIMALITY

The problem is linear: $I(x) = I \cdot x$  $S(y) = S \cdot y$  $I, S$ — matrixes $n \times m$ and $p \times n$.

Optimal and central algorithms are unknown.

Projective estimation $x_{pr}$ is a solution of the least modules (LM) problem

$$x_{pr} = \arg \min_x \left( \| I \cdot x - y \| = \sum_q |y_q - i'_q \cdot x| \right) \quad q = 1, 2, \ldots, n$$

which can be reduced to linear programming problem.

The estimation $x_{pr}$ is interpolated and it is not central.
TWO DIMENTIONAL CASE \((m=2)\)

\[ y_q = x_1 + (q-n/2) \cdot x_2 + \delta_q \quad |\delta_q| \leq \delta \quad q=0,1,...,n \]

\[ z = x \quad U_x(y) = \cap_q \{ y_q - \varepsilon \leq x_1 + (q-n/2) \cdot x_2 \leq y_q + \varepsilon \} \]

central estimation

\[ x_{k,m} = \min_{x \in \mathcal{U}_x} x_k \quad z_{k,M} = \max_{x \in \mathcal{U}_x} x_k \quad y_q - \varepsilon \leq x_1 + (q-n/2) \cdot x_2 \leq y_q + \varepsilon \quad k=1,2 \quad q=0,1,...,n \]

projective estimation

\[ \min \alpha \quad y_q - \alpha \cdot \varepsilon \leq x_1 + (q-n/2) \cdot x_2 \leq y_q + \alpha \cdot \varepsilon \quad \text{in space} \ (x_1, x_2, \mathcal{U}) \quad q=0,1,...,n \quad 0<\alpha \leq 1 \]

The ratio of mean errors of GA and LS estimations (simulation \(-1000)\)

<table>
<thead>
<tr>
<th>n</th>
<th>parameter (x_1)</th>
<th>parameter (x_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>uniform distrib.</td>
<td>gaussian-3(\sigma) distrib.</td>
</tr>
<tr>
<td></td>
<td>(\delta x_{1,\text{en}} / \delta x_{1,\text{LS}})</td>
<td>(\delta x_{1,\text{en}} / \delta x_{1,\text{LS}})</td>
</tr>
<tr>
<td>10</td>
<td>0.74 0.80</td>
<td>1.20 1.39</td>
</tr>
<tr>
<td>20</td>
<td>0.50 0.64</td>
<td>1.41 1.69</td>
</tr>
<tr>
<td>30</td>
<td>0.52 0.56</td>
<td>1.55 1.85</td>
</tr>
<tr>
<td>50</td>
<td>0.39 0.43</td>
<td>1.86 2.31</td>
</tr>
<tr>
<td>100</td>
<td>0.30 0.34</td>
<td>2.43 2.99</td>
</tr>
</tbody>
</table>
Real values: $x_1=10.0$, $x_2=1.0$

Reliaizations of LS and central estimations for model $m=2$
(distribution of error measurements: Gauss truncated of the level of $3\sigma$, $n=100$)
Real values: $x_1=10.0$, $x_2=1.0$

Relizations of LS and central estimations for model $m=2$
(distribution of error measurements: triangle, $n=100$)
Real values: $x_1=10.0$, $x_2=1.0$

Relaxations of LS and central estimations for model $m=2$
(distribution of error measurements: uniform, $n=100$)
The comparison of algorithms used two different approaches (real situation, simulation)

SCENARIO:
· 1-1 — one passage of SO through one radar coverage zone
· 1-N — all passages of SO through one radar coverage zone during 10 days
· 3-N — all passages of SO through three radars coverage zone during 10 days.

INITIAL DATA:
· coordinates of radar stand points:
  1: \( \gamma = 0.7, \theta = 0 \)  2: \( \gamma = 1.0, \theta = 0.6 \)  3: \( \gamma = 0.75, \theta = -0.6 \)  \( h = 0 \).
· radar coverage zone: \( 5^\circ \leq \gamma \leq 60^\circ \), \( \gamma \) — elevation angle.
· measurement times:
  1-1: each 5 sec from SO entry into radar coverage zone up to exit, 1-N,3-N: each 5 sec during 50 sec
· measured parameters:
  the spherical coordinates \( d, \varphi, \theta \) — range, azimuth, elevation angle.
· measurements errors:

the distribution of uncorrelated errors: uniform, triangular, Gaussian truncated at the level of three "sigmas"; root-mean square value of errors $d = 0.15$ km, $\xi = 0.003$ radians, $\eta = 0.003$ radians; systematic distance error is equal to half the maximum value of the uncorrelated error with arbitrary sign.

· motion model:

Earth - EGM-96 $i_{gg} = 75$, Moon, Sun, atmospheric perturbations, solar radiation pressure.

· orbit parameters: 6-vector of Lagrange elements in the moment $t_n$.

· solution parameters: the coordinates of position vector propagated to $n_d = 0, 1, 3, 5, 7, 10$ days in the first orbital coordinate system $(r, t, n)$.

· ratio of area to SO mass: is a constant and equal to 0.01 m$^2$/kg.

· SO trajectories: eccentric $e = 0$, inclination $i = 57^\circ$, altitudes above ground $h = 800$ km and 1500 km.

· number of measurements: $n = 65-155, 550, 1750$ in 1-1, 1-N, 3-N.

· number of realization in simulation: $n_r = 100$.

COMPARATING INDEX OF ALGORITHMS: $\delta_{\xi} = (\sum_i |\xi_i - \xi_i|)/n_r$ - mean error
1 radar, 1 passage, uncorrelated errors

$\delta \xi(X_{LS}) \text{ [m,m/s]}$ and $\delta \xi(X_{CN})/\delta \xi(X_{LS})$ for various measurement error distributions

<table>
<thead>
<tr>
<th></th>
<th>1al trajectory</th>
<th>1ac trajectory</th>
<th>2al trajectory</th>
<th>2ac trajectory</th>
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<tr>
<td>$d$</td>
<td>18</td>
<td>0.3</td>
<td>1.2</td>
<td>1.4</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>690</td>
<td>0.3</td>
<td>1.0</td>
<td>2.0</td>
</tr>
<tr>
<td>$\beta$</td>
<td>630</td>
<td>0.2</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>$\dot{d}$</td>
<td>0.09</td>
<td>0.3</td>
<td>1.0</td>
<td>1.8</td>
</tr>
<tr>
<td>$\dot{\alpha}$</td>
<td>2.70</td>
<td>0.4</td>
<td>1.0</td>
<td>1.6</td>
</tr>
<tr>
<td>$\dot{\beta}$</td>
<td>0.42</td>
<td>0.4</td>
<td>1.0</td>
<td>1.6</td>
</tr>
</tbody>
</table>
1 radar, all passages during 10 days, uncorrelated errors

\[ \delta \xi(Z_{LS}) \text{ [m]} \] and \[ \delta \xi(Z_{CN})/\delta \xi(Z_{LS}) \] for various measurement error distributions

| \( n_d \) | LS [m] | uniform | | | | triangular | | | | gaussian-3\( \sigma \) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | \( r \) | \( t \) | \( n \) | \( r \) | \( t \) | \( n \) | \( r \) | \( t \) | \( n \) | \( r \) | \( t \) | \( n \) |
| 0 | 34 | 54 | 21 | 0.17 | 0.10 | 0.22 | 1.0 | 0.5 | 1.0 | 2.1 | 1.0 | 2.0 |
| 1 | 34 | 54 | 24 | 0.17 | 0.13 | 0.22 | 1.0 | 0.7 | 1.0 | 2.1 | 1.3 | 2.0 |
| 3 | 34 | 39 | 29 | 0.17 | 0.21 | 0.22 | 1.0 | 1.0 | 1.0 | 2.1 | 2.2 | 2.0 |
| 5 | 34 | 35 | 34 | 0.17 | 0.29 | 0.22 | 1.0 | 1.2 | 1.0 | 2.1 | 2.4 | 1.9 |
| 7 | 35 | 44 | 40 | 0.17 | 0.25 | 0.22 | 1.0 | 1.1 | 1.0 | 2.1 | 2.2 | 1.8 |
| 10 | 35 | 73 | 47 | 0.02 | 0.19 | 0.22 | 1.0 | 0.9 | 1.0 | 2.1 | 1.3 | 1.8 |
3 radars, all passages during 10 days, uncorrelated errors
\( \delta \xi (Z_{LS}) \) [m] and \( \delta \xi (Z_{CN})/\delta \xi (Z_{LS}) \) for various measurement error distributions

<table>
<thead>
<tr>
<th>( n_d )</th>
<th>LS [m]</th>
<th>uniform</th>
<th>triangular</th>
<th>gaussian-3( \sigma )</th>
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<tbody>
<tr>
<td>0</td>
<td>r</td>
<td>t</td>
<td>n</td>
<td>r</td>
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<tr>
<td>31</td>
<td>27</td>
<td>6</td>
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<td>0.05</td>
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<tr>
<td>32</td>
<td>59</td>
<td>11</td>
<td>0.02</td>
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</table>
3 radars, all passages during 10 days, uncorrelated and systematic errors

\[ \delta \xi (Z_{LS}) \text{ [m]} \quad \text{and} \quad \delta \xi (Z_{CN})/\delta \xi (Z_{LS}) \] for various measurement error distributions

<table>
<thead>
<tr>
<th>( n_d )</th>
<th>( LS \text{ [m]} )</th>
<th>( r )</th>
<th>( t )</th>
<th>( n )</th>
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<th>( \text{triangular} )</th>
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</tr>
<tr>
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<td>36-40</td>
<td>100-180</td>
<td>31-54</td>
<td>0.06</td>
<td>0.11</td>
<td>0.10</td>
<td>0.4</td>
</tr>
<tr>
<td>7</td>
<td>37-40</td>
<td>105-180</td>
<td>41-71</td>
<td>0.06</td>
<td>0.12</td>
<td>0.09</td>
<td>0.4</td>
</tr>
<tr>
<td>10</td>
<td>38-41</td>
<td>110-185</td>
<td>55-99</td>
<td>0.06</td>
<td>0.13</td>
<td>0.08</td>
<td>0.4</td>
</tr>
</tbody>
</table>
3 radars, all passages during 10 days, uncorrelated errors, $\tilde{\varepsilon} = 2\varepsilon$

$\delta\zeta(Z_{LS})$ [M] and $\delta\zeta(Z_{CN})/\delta\zeta(Z_{LS})$ for various measurement error distributions

<table>
<thead>
<tr>
<th>$n_d$</th>
<th>LS [m]</th>
<th>uniform</th>
<th>triangular</th>
<th>gaussian-3$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n</td>
<td>r</td>
<td>t</td>
<td>n</td>
</tr>
<tr>
<td>0</td>
<td>31</td>
<td>27</td>
<td>6</td>
<td>0.6</td>
</tr>
<tr>
<td>1</td>
<td>31</td>
<td>20</td>
<td>6</td>
<td>0.6</td>
</tr>
<tr>
<td>3</td>
<td>32</td>
<td>15</td>
<td>7</td>
<td>0.6</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
<td>23</td>
<td>8</td>
<td>0.6</td>
</tr>
<tr>
<td>7</td>
<td>32</td>
<td>36</td>
<td>9</td>
<td>0.6</td>
</tr>
<tr>
<td>10</td>
<td>32</td>
<td>59</td>
<td>11</td>
<td>0.6</td>
</tr>
</tbody>
</table>
1 radar, all passages during 10 days, uncorrelated errors with uniform distrib. 
\[ \frac{\delta \zeta(Z_{LS,igg})}{\delta \zeta(Z_{LS, igg=75})} \] and 
\[ \frac{\delta \zeta(Z_{CN,igg})}{\delta \zeta(Z_{CN, igg=75})} \] with \( i_{gg} = 8, 16, 32 \)

<table>
<thead>
<tr>
<th>coordinates</th>
<th>LS estimation</th>
<th>central estimation (CN)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( r )</td>
<td>( t )</td>
</tr>
<tr>
<td>model ( i_{gg} )</td>
<td>8 16 32</td>
<td>8 16 32</td>
</tr>
<tr>
<td>( n_d = 0 )</td>
<td>3.5 0.9 1.0</td>
<td>1.0 2.0 0.9</td>
</tr>
<tr>
<td>( n_d = 1 )</td>
<td>3.7 0.9 1.0</td>
<td>5.0 1.2 0.9</td>
</tr>
<tr>
<td>( n_d = 3 )</td>
<td>2.9 0.9 1.0</td>
<td>9.2 2.2 1.1</td>
</tr>
<tr>
<td>( n_d = 5 )</td>
<td>4.1 1.2 1.0</td>
<td>5.6 1.3 1.0</td>
</tr>
<tr>
<td>( n_d = 7 )</td>
<td>5.7 1.2 1.0</td>
<td>8.0 1.2 1.0</td>
</tr>
<tr>
<td>( n_d = 10 )</td>
<td>5.0 1.3 1.0</td>
<td>4.1 3.0 0.9</td>
</tr>
</tbody>
</table>
THE CONCLUSIONS

1. The non-statistical approach corresponds to properties of real measurements errors.
2. The non-statistical approach provides the computation of the solution parameters errors corresponding to their real values.
3. The interpolated algorithms of the non-statistical approach have property of adaptation to real measurements errors.
4. The estimation of solution parameters in interpolated algorithms turns out the more precisely and advantage in accuracy in comparison with method LS the more, than the big part of errors is concentrated in near-border area.
5. The problem is more informative, the central algorithm in relation to LS better works.
6. The correlations in measurement errors worsens quality of solution parameters estimation in central algorithm less, than for method LS.
7. The projected algorithm is not critical to accuracy of knowledge of errors upper limit. The central algorithm has not such feature. The projected algorithm can lose in accuracy to the central algorithm no more than in 2 times.
8. The central and projected algorithms are more critical to methodical errors of SO movement model, than the LS method.

WHEN AND HOW TO APPLY ALGORITHMS OF NON-STATISTICAL APPROACH IN PRACTICE?
The answer to this question depends on error characteristics of real measuring tools.
Comparison of LS and central estimations for parameter $r$ of the orbital coordinate system

(Iridium 33 and Cosmos 2251 collision, February 10, 2009, 16:56)
\[ p_c = k \exp(-0.5k_{rr}) \]
\[ k = S \sqrt{\sum \sum v^2} (4 \sum \sum k_{vv} \det K_1 \det K_2) \]
\[ \left( K_1^{-1} + K_2^{-1} \right)^{-0.5} \]
\[ S = \frac{(d_1 + d_2)^2}{4} \]
\[ k_{rr} = \sum \sum r \left( K_1 + K_2 \right)^{-1} \sum \sum r' \]
\[ k_{vv} = \sum \sum v \left( K_1 + K_2 \right)^{-1} \sum \sum v' \]

\[ t_{\min} \] — the time when two objects approach the minimum distance; \( \sum \sum r, \sum \sum v \) — vectors of the relative position and velocity in \( t_{\min} \); \( K_1, K_2 \) — covariance matrices of the errors of determination of the positions of both objects for the time \( t_{\min} \); \( d_1, d_2 \) — dimensions of the objects.

\[ p_c \approx k \exp(-A) \]
\[ k = \frac{(d_1 + d_2)^2}{\left( \sigma_u \cdot (11\sigma_u \sin^2 \alpha + 16\sigma_w \cos^2 \alpha) \right)} \]
\[ A = (\delta u)^2 / (4\sigma_u^2) + (\delta v)^2 / (2\sigma_v^2) + (\delta w)^2 / (2\sigma_w^2 \cos^2 \alpha + 2\sigma_v^2 \sin^2 \alpha) \]
\[ \sum \sum u, \sum \sum v, \sum \sum w \] — the projections of \( \sum \sum r \) in the directions \( u, v, w \) of the orbital coordinate system of an object in a near-circular orbit; \( \sum \sum \) — angle between the object velocity vectors \( \sum \sum v_1 \) and \( \sum \sum v_2 \) at time \( t_{\min} \).

It follows from (1) and (2) that if a collision occurs in this dangerous approach

- if the errors in the determination of the approaching SO positions decrease, then \( p_c \) increases. For example, if the errors decrease by an order of magnitude, \( p_c \) increases by two orders of magnitude;
- overestimation or underestimation of the calculated errors result in a significant decrease in \( p_c \).

For example, if the calculated errors are increased by an order of magnitude, \( p_c \) decreases by two orders of magnitude and it tends to zero if errors are decreased by an order of magnitude.