DEVELOPMENT OF OPERATIONAL MODAL ANALYSIS
TECHNIQUES FOR LAUNCH DATA

George James*

Correlation-based Operational Modal Analysis (OMA) has been used to act as a pathfinder for the development of techniques to process launch data. This activity, which emphasizes the estimation of modal damping, operates in an extremely challenging environment due to the rapidly changing modal properties, loading conditions, and vehicle configuration as well as the limited response data that is available. A historical review of these efforts is provided. Recent work with launch environment data has identified two issues for immediate attention: beat-like phenomena in the correlation functions of short time records and potential interactions between the vehicle control systems and extracted damping in the lowest bending modes of the vehicles. A working hypothesis and mitigation for the first problem (beat-like phenomena) has been developed but a convergence metric was needed. This paper provides a first generation metric to allow the development to continue. The second problem of potential control system interactions is still being scoped and has become the near-term driver for the development of higher-fidelity tools (as the beating mitigation).

INTRODUCTION

The spacecraft launch environment is a highly complex and non-stationary event that is characterized by high amplitude input forces, highly variable loads, a wide spectrum of responses, constantly changing vehicle mass, active control interactions, staging, and limited instrumentation. At the same time, structural response analyses and loads estimations must be performed with models that are only partially validated using ground test data due to the fact that access to diagnostic and environmental ground tests are limited. To compound matters, project managers tend to reduce uncertainty factors designed to protect for loads increases and model unknowns. As a result, the designs progress rapidly before loads and structural problems are uncovered. This means that there are very few tools available to recover from structural dynamics issues in such a highly dynamic environment without costly redesigns late in the design cycle or in early operations.

Traditional modal testing applies a known input to the structure (or components/test articles) and structural dynamic parameters are then extracted. In spite of the relatively pristine data, these dedicated tests are often difficult to schedule or afford (and rarely achieve flight-like loading conditions). The inclusion of operational modal testing and analysis tools can be used to offset these limitations by providing additional cost and schedule effective opportunities for diagnostic information extraction. These opportunities are available on the ground and during flight as well as on

* Aerospace Engineer, Loads and Structural Dynamics Branch, NASA Johnson Space Center, Houston, TX, 77058.
full-up systems, subsystems, components, and test articles. Therefore data from early test flights and early operational flights become important resources for producing diagnostic information.

**TECHNICAL BACKGROUND**

The technology for extracting structural dynamic properties from structures (i.e. modal testing) has rapidly improved over the last several decades. Operational modal analysis (OMA) is one of the specialized spin-offs that have grown in importance over the last 20 years. In OMA, a known and measured input is not artificially imparted to the structure to drive the known responses but measurements are made in-situ and processed to obtain a subset of the desired modal data. This approach is very useful for large in-service structures that cannot be removed from service effectively (e.g. bridges, buildings, wind turbines, off-shore structures, etc.). One of the earliest OMA techniques was the Natural Excitation Technique (NExT). The development showed that for a class of inputs, the auto and cross-correlation functions could be processed as time decay functions to estimate the modal frequencies and modal damping properties. Time domain estimators, such as Polyreference or the Eigensystem Realization Algorithm (ERA) have been used to process such data. Appendix A contains the theoretical background for this technique. Recent years have seen the OMA field as become rich with other advanced techniques with broad applicability. However, NExT has continued to spawn on-going efforts to improve, expand, and further understand the approach.

**Alternative Approaches for Traditional OMA**

There are two general classes of algorithms for performing stationary linear OMA: (1) time history-based techniques that are generally related to Stochastic Subspace Identification (SSI) and frequency domain-based techniques that are related to Frequency Domain Decomposition (FDD). Early time domain approaches included tools like the Random Decrement and Maximum Entropy Methods. The technical basis for the NExT approach mentioned previously involved converting measured responses into auto and cross-correlation functions and processing with standard time domain modal analysis routines. However, the more general SSI techniques directly integrate the correlation calculations and modal processing algorithms into a single step rooted in discrete time system identification theory. The earliest manifestations of FDD were peak picking and half-power bandwidth estimation schemes operating on the Power Spectral Density (PSD) functions. However the advanced FDD algorithms refine the modal parameter estimates using powerful tools like the Singular Value Decomposition (SVD). An interesting direction for frequency domain approaches involves the use of Hilbert transforms applied to PSD’s to obtain biased Frequency Response Function (FRF) estimates.

There is another direction in operational testing that involves estimating the forces acting on the system. This would allow more traditional FRF-based approaches to be used for system identification. These forces can be estimated via known mass changes to the system or via hybrid analytical/experimental data. For non-stationary systems, Wavelet Analyses represents one possible approach. Another possible approach is via the Wagner-Ville developments. For non-linear and non-stationary systems, the empirical Hilbert-Huang method is a possibility.

**Launch Environment Analyses**

There have been a limited number of reported attempts to analyze flight data to extract modal parameter information, although there are certainly many other unreported attempts. The time domain approaches based on correlation and SSI are generally used for flight data analyses as the rapidly changing vehicle properties do not allow the full advantages of the frequency-domain approaches to be realized. The responses are generally broken into a series of time windows, each of short duration (and quite often significant overlap), that are processed individually. If the load-
ing and system characteristics are fairly constant over each window, then estimates of the changing parameters can be obtained as a function of flight time. \(^{28,29,30,31,32,33,34,35}\)

Three of the references listed above show that one trajectory for NExT has been to act as a pathfinder for the development of operational analysis techniques to process launch data. \(^{28,29,30}\) This effort is doubly challenging as modal damping is one of the most sought after parameters from the launch environment, which is difficult to extract even in well controlled stationary environments. However, flight damping during launch becomes a critical part of the discussions during the design and operations of space vehicles due to the control over the response of structures and components. Hence, modal damping has become an important metric for the utility flight data and flight data analyses.

Reference \(^{28}\) discusses the earliest work in the application of the NExT/OMA approach to launch data from a missile-based system. The work used tight narrow-band filters to limit the data under processing to one or two modes at time. Also, the time windows generally covered two to four cycles of the modes and had significant overlap. In order to make this analysis tenable, the modal engine (ERA\(^3\)) had to be called automatically. Even with tight filtering and the processing software able to call ERA when needed, this analysis was a very labor intensive process. The manual effort involved assessing the results of the processing for each window, making decisions on the selected roots, resetting parameters if required, and restarting the processing when needed. However, the traces for modal frequency and damping look fairly reasonable and smooth (except for the first mode damping in first stage flight).

Reference \(^{29}\) discusses a later launch analysis from the Space Shuttle. For this analysis and automation process called AUTO-ID was added to the tool. \(^{36,37}\) The addition of this technique eased the computational burden of extracting parameters in a consistent manner from the multitude of correlation functions calculated from the sliding time window segments of the random time histories to allow a rapid assessment of the data. This was a much less labor-intensive process that the original processing effort as discussed in Reference \(^{28}\). As a result many more modes were assessed in much less time than seen in the previous case study. However, the results were not a pristine and more excursions in the frequency and damping were allowed as a result. The reported data for modal damping still shows trends and excursions during flight.

Reference \(^{30}\) is a recent study performed on the PA-1 test flight. This test flight provided a very challenging data set with a very short flight time and extremely rapidly changing modal frequencies. Autonomous identification was not used and the amount of frequencies studied was less that the study discussed in Reference \(^{29}\). The user effort required was intermediate between the two previous studies discussed. Since the user interacted with data to a greater extent than previously and the experience of the previous exercises was available, two problems were for immediate attention: beat-like phenomena in the correlation functions of short time records and potential interactions between the vehicle control systems and extracted damping in the lowest bending modes of the vehicles. A working hypothesis for the first problem (beat-like phenomena) has been developed in which the phenomena are apparent increases in correlation due to the lack of ability to temporally average out the random characteristics of the responses. A limited number of potential mitigations for this effect are in-hand as well plans to assess other non-correlation based approaches. However, the second problem of potential control system interactions is still being scoped. A database of analytical and flight data from the recent Ares 1-X launch is available to study the effects of control system interactions with in-flight extracted modal properties. The continuing effort to understand and then develop and implement mitigations for the first of these two issues is the primary focus of this paper.
LAUNCH ANALYSIS ISSUE: CORRELATION BEATING

A most significant complexity associated with operational analysis of launch systems is the unsteadiness due to rapidly changing mass properties. This usually drives the available time records to be very short due to the need to utilize some type of sliding window analysis (at least for a process that assumes stationarity). The analysis of several recent data sets has shown that one effect (at least on the correlation-based processing approaches) is a “beating” or “blooming” phenomena which limits the amount of the correlation functions that can be used for processing. Although the first low-lag points in the correlation functions are relatively unaffected, the higher-lag time data points are relatively useless for analysis.

Previous Findings

Figure 1 illustrates the beating phenomena as illustrated by a simple one DOF analytical model of a 10 Hz mode with 1% damping excited by random white noise using Newmark-Beta integration. The 32,768 length time history has a time step of .001 seconds. The random input is shown in the top plot of Figure 1. The displacement response of the 10 Hz system is shown in the middle plot. Displacement is used as opposed to the more easily measured acceleration as it illustrated the issue with more clarity. Notice that the response shows random excitations of the 10 Hz system mode, which eventually damps out. The lower plot provides the autocorrelation function of the displacement shown in the middle plot. The beating phenomena are clearly seen as the correlation increases at longer lags. Note that the “beating” terminology is adopted as the correlation function looks like a time history of closely spaced modes interacting or “beating”. The periodic increases in correlation deviates from the theoretical damped sinusoids that are expected from OMA/NExT correlation functions and limit the utility to separate closely spaced modes as only early-lags can be used for analysis. The working hypothesis for this phenomena is that the random “blooms” in the response data (middle plot of Figure 1) result as the internal modes are randomly excited by the input. During the correlation process these “blooms” in the response data become the beating phenomena in the correlation functions (see bottom plot of Figure 1).

Previous work to address this issue suggested that the beating effects can be reduced via repeated correlation calculations using the same parent time data. Figure 2 shows the effectiveness of this approach. The top plot shows an autocorrelation function of a 10 Hz single DOF system excited randomly. If another correlation calculation is performed using the first autocorrelation function as the parent data then the correlation function shown in the middle plot results. The bottom plot results after performing an additional eight correlation calculations using the function shown in the middle plot as the parent data (10 correlation calculations total). This obviously produces a damped sinusoidal function as expected. It can be shown that the proper damping does result after a number of these correlations are performed. However, the application of additional correlations does numerically alter the extracted damping. Hence, this approach needs a convergence metric to allow the analyst to know when to stop performing additional correlations. An initial draft of such a metric is presented in this paper.

Development of a Convergence Metric

The problem at hand involves performing additional specialized averaging (in the form of autocorrelation functions) on the randomly excited data. The desired result is a mathematical function that more closely matches the theoretical expectation of damped sinusoids. The primary issue in the current single pass correlation functions is not the sinusoidal content but the incomplete capture of the decaying exponential envelope. Hence, the first step will be to extract the envelope. To do this we start by generating an analytic function using the Hilbert transform of the first pass correlation functions (either an auto or a cross-correlation function):
Figure 1. Beating Phenomena in Correlation Functions in Single DOF Analytical Data

Figure 2. Use of Repeated Correlation Calculations to Reduce Beating Phenomena
\[ C_{ij}(t) = R_{ij}(t) + \sqrt{1 - \tau^*} H_{ij}(t), \] where

\[ R_{ij} \] is correlation function between outputs \( i \) and \( j \);
\[ H_{ij} \] is the Hilbert transform of \( R_{ij} \); and
\[ C_{ij} \] is the analytic function associated with \( R_{ij} \) and \( H_{ij} \).

The Hilbert transform is defined as follows:

\[ H_{ij}(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{R_{ij}(\tau)}{t - \tau} d\tau. \] (2)

The envelope is then the amplitude of the complex analytic function:

\[ A_{ij}(t) = \sqrt{R_{ij}^2(t) + H_{ij}^2(t)}. \] (3)

Figure 3 contains an example of the previous functions. The top plot is the numerical displacement response of a single-mode 10Hz system with 1% damping as excited with random inputs. The middle plot shows the autocorrelation function from the data in the top plot. The bottom plot shows the associated envelope as described by equation (3) above. Now the theoretical result suggests that the correlation function looks a decaying sinusoid, hence for this single-mode system, the envelope should look like an exponential decay:

\[ A_{ij}(t) \approx A_0 e^{-at}. \] (4)

Therefore for this simplified system, we can take the natural log to simplify:

\[ \ln(A_{ij}(t)) \approx \ln(A_0) + \ln(e^{-at}) = a_0 - at. \] (5)

There are two possible approaches to determine estimates of the exponential parameter from the log envelope. The first approach is to fit a line to the natural log of the envelope with the offset providing \( a_0 \) and the slope \( a \) parameter. For subsequent discussions, this approach shall be referred to as the “linear slope” approach and the slope parameter designated as “\( a_1 \)”. The alternative approach is to take a mean value of the differences between any two values of the log envelope and divide by the time step. This approach will be referred to as the “mean difference” and the slope parameter designated as “\( a_2 \)”. Figure 4 shows those two approaches and the best fit estimates for the first half of the records of the log envelope given in Figure 3.

In order to further increase the content of the modal response and reduce the effects of the random forcing function, the current correlation function is subjected to another pass through the correlation processing step. Correlation processing (in a time domain sense) involves a summation process of all data separated by the same number of time steps (or correlation lags in this case):

\[ R_{ij}(\tau) = \sum_{t=0}^{t_{\max} - \tau - 1} x(t + \tau) y(t) \quad \tau \geq 0 \]
\[ R_{ij}(\tau) = R_{ij}(-\tau) \quad \tau < 0 \] (6)
Figure 3. Envelope of Correlation Function of a Single DOF Analytical Data

Figure 4. Fits to Log Envelope of a Single DOF Analytical Correlation Function
Now if we substitute the original time history products for the current correlation function and add a subscript to denote the iteration or number of the successive correlation steps (k):

\[ R_{ijk}(\tau) = \sum_{t=0}^{t_{\text{max}}-\tau-1} R_{ijl}(t + \tau)R_{ijl}(t) \quad \tau \geq 0 \]

\[ R_{ijk}(\tau) = R_{ijl}(-\tau) \quad \tau < 0 \]  \hspace{1cm} (6)

Where \( l = k - 1 \).

Hence, the variables of interest will be estimated at each iteration and will receive a “k” subscript: \( H_{ijk}(t) \), \( C_{ijk}(t) \), \( A_{ijk}(t) \), \( a_{0k} \), \( a_{1k} \), and \( a_{2k} \). The slope parameters are updated each iteration and compared to the previous value and scaled with respect to the first slope value to produce a convergence metric (labeled as \( b_{0k} \) and \( b_{1k} \)). Hence for the linear slope metric:

\[ b_{1k} = 100 \times \left( \frac{a_{1k} - a_{11}}{a_{11}} \right) \]  \hspace{1cm} (8)

And for the mean difference metric:

\[ b_{2k} = 100 \times \left( \frac{a_{2k} - a_{21}}{a_{21}} \right) \]  \hspace{1cm} (9)

Figure 5 shows typical convergence histories for these two metrics. The top plot provides the actual values of the slope parameters (as illustrated in Figure 4) for multiple subsequent correlation iterations. The lower plot shows the traces of the related convergence metrics. For this example the process was ended when the convergence metric was lower than .1% in either parameter. Typically the mean difference converges first. Figure 6 shows the fits to the converged log envelopes. Although there are some obvious numerical issues for longer lag times, the overall trends are much closer to the expected exponential decay model. Figure 7 shows the final correlation trace after convergence in the top plot. The middle plot contains the final envelope of the correlation trace. These plots show the expected decaying exponential shape. After performing a modal identification on the data shown in Figure 7, the resulting modal frequency is found to be 10.1 Hz with .99% damping. The resulting synthesis to the converged data is provided in Figure 8. The final extracted frequency and damping is used to generate damped sine and cosine functions which are least-squares fit to the correlation data to generate the synthesis.

**Application of Convergence Metric to Field Data**

There are multiple aspects of the resulting approach of nested correlation functions and the associated convergence metric. These issues include such topics as multiple modes, short time records, changing frequencies, time step, etc. However, the next step undertaken was to assess the application of this procedure to real data as an early sanity check. The data used was lander test data as was discussed in Reference [38].
Figure 5. Convergence of a Multiple Correlations of Single DOF Analytical Data

Figure 6. Converged Fits to Log Envelope of Single DOF Analytical Data
Figure 7. Envelope of Converged Correlation Function of Single DOF Analytical Data

Figure 8. Data and Synthesis of Modal Fit to Converged Correlation of Analytical Data
The RR1 vehicle is a lander test craft built by Armadillo Aerospace. It was flown in a tethered hover flight for over 30 seconds to produce the data set used herein (see Figure 9). The RR1 flight data was unique in the respect that it was real flight data with high amplitude excitation, an active control system, and mass changes due to fuel use. However, this data set did not contain significant changes in natural environments or forward velocity effects as it was a hover test. It was also found that the modal frequency changes were not significant over this flight time and the data could be assumed to be stationary.

The RR1 data set consisted of 31,000 samples of three low frequency accelerometers that were sampled at 1000 samples per second and useful for 1.4 to 350 Hz. The channels were ranged for +/- 16g with a .007g resolution. There was a very strong mode in the 28Hz region. One sensor was strong in this mode and that sensor was used in the exercises below. The data was filtered with a 4th order Butterworth filter between 10 and 45Hz using the MATLAB “filtfilt” command. Hence, this data was chosen purposely to exercise the convergence metric on real-world data that was as similar to the simple analytical data as possible. The similarities included a similar data length, a similar sample frequency, apparent stationarity of the modal parameters, and a response dominated by a single dynamic phenomenon.

![Figure 9. RR1 Vehicle in Static and Tethered Flight Configurations](image)

Figure 10 and Figure 11 provide all 31000 samples of the single RR1 sensor data in the top plots. In Figure 10, the frequency domain representation is provided in the other plots (linear and semi-log versions). Notice that the data content is strong around 28 Hz but the peak is not clean due to the random excitation of the environment. The second plot of Figure 11 provides the resulting autocorrelation function, and the final plot of Figure 11 is the envelope of the correlation function as derived using the process from the previous section. Figure 12 provides the slope parameters and fits to the log of the envelope to estimate the slope parameters after this first correlation step. After performing multiple correlation iterations, the slope parameters vary and converge as shown in Figure 13. The trends seen in this convergence plot of a local minimum between 5 and 10 iterations and significant minimum in the mean difference trace around 30 iterations appears quite often in both the numerical and the experimental data analyses to-date. Figure 14 provides the clean decaying exponential envelope at the end of the iterations. Figure 15 provides the data in time and frequency domains after the correlation iterations have converged. The synthesis using the estimated modal frequency and modal damping parameters is also provided but overlays and occludes the correlation data.
Figure 10. RR1 Vehicle Sensor Data in Time and Frequency Domain Representations

Figure 11. RR1 Vehicle Input Data, Original Correlation, and Envelope
Figure 12. Fits to Log Envelope RR1 Vehicle Correlation Function

Figure 13. Fits to Log Envelope RR1 Vehicle Correlation Function
Figure 14. Envelope of Converged Correlation Function of RR1 Vehicle Data

Figure 15. Data and Synthesis of Modal Fit to Converged Correlation of RR1 Data
Effects of Data Length

The data provided above to study and develop a mitigation strategy for correlation beating on short time records is subject to several simplifying factors that will have to be removed before the process becomes truly useful. A major factor is the need to study the effects of multiple dynamic phenomena with significant contributions in a time record. Another factor is the need to understand the limits of data stationarity. The effects of multiple sensor processing will need to be studied but is not expected to challenge the process significantly. A more complete development of the effects of numerical round-off error and system noise is called for as well. Other secondary factors are all in play at this early development stage. These include: (1) the role that the phase of the analytic function may play, (2) digital parameter shifting (as seen in numerical integration algorithms), and (3) the order to apply the cross-correlation and iterative autocorrelations when using data from different sensors. However, the most critical unknown to be removed before this mitigation process is useful for short time record launch data analysis are the actual effects of data record length. Hence, a scoping study of the effects was performed and is included herein.

This scoping study involved utilizing several different data record lengths of data to develop estimates of the modal frequency and modal damping to assess the trends in the parameters. For the 10Hz analytical data, data lengths from 1024 samples to the full 32768 length data record were assessed in increasing data record length increments of 1024 samples for 32 different analyses. Figure 15 provides the results of this study. The top plot shows the variation of the extracted frequency as the data record length processed in increased. The variations are exaggerated due to the scale and are not significantly problematic. However, the lower damping plot shows a clear trend of increasing damping estimates as the record length is increased. This is an issue that must be dealt with as damping is a critical parameter in these studies and the shorter time records are the region that launch data processing will be utilizing. The good news is that the process breakdown does not appear to be stochastic but has a clear trend and is deterministic. Hence, it may be possible to eliminate, reduce, or correct for the low record length trends after understanding the root cause.

The RR1 data was subjected to the same study and similar trends result. The benchmarks for data comparison in Figure 17 are the final values for estimated frequency and damping from the analysis of the full 31000 sample data set. The analyses were performed in increments of 1000 samples starting at 1000 samples or one second of data. The frequency variations are noticeable but seem manageable. The damping estimates do increase significantly for short record lengths and the effect must be dealt with. The initial attack on this issue will involved a complete understanding of the numerical effects of multiple correlation functions. This includes well-known correlation bias effects that may be exerting an influence at these low sample count calculations. Also, an understanding of the nature of the convergence process developed herein will be required. Other approaches based on more advanced signal processing schemes, force reconstruction/transfer function techniques, and a more complete understanding of the content of the correlation envelope will be used as needed.

LAUNCH ANALYSIS ISSUE: CONTROL SYSTEM EFFECTS

The ability to process launch data is allowing a more complete understanding of the vehicle response and effects of the environment on the structural loads. A significant aspect of that understanding involves accounting for the control-structure effects in flight – especially first stage flight with active aerodynamic, wind, and maneuvering disturbances. Previous data processing studies have seen potential effects of this interaction between the control system and the modal responses – primarily frequency and damping of the lowest bending modes. Each of these
Figure 16. Effect of Data Record Length using Single DOF Analytical Data

Figure 17. Effect of Data Record Length using RR1 Vehicle Data
past activities extracted fairly significant deviations in damping of the low-frequency modes during first stage flight. In Reference [28], a significant excursion from 1% damping to 2% damping on the first mode of 12 Hz was seen at 20 seconds into first stage flight. After this the damping continued to oscillate by .5% as it settled into the expected range of .5% to 1%. Taken as a stand-alone analysis, these variations are most likely an effect of the difficulty of extracting damping in a challenging situation of this large missile system during launch.

In Reference [29], analyses of Space Shuttle data from STS-105 and STS-108 had examples of low frequency modes at 4Hz and 3Hz with fairly large excursions of damping from 3% to 5%. Given the limited number of frequency/damping pairs that could be reasonably extracted from this data, with the techniques in use at the time, the error band around these damping values is fairly large (especially if taken out of context of the other analyses). Reference [30] used data from a launch abort system test flight to further exercise these tools. Data from this mission showed a rise from 2% damping up to 6% and then falling back to 2% damping in the 7Hz mode of the system. The frequency was increasing as expected but did not show a significant anomalous variation during this damping excursion. Reference [33] was performed by a distinctly different group using different tools that the three previous studies. The Ariane V data showed damping variations with similar trends to those seen in the three previous references; however the scale of the data was unavailable.

All of these results taken together suggest that there are emergent conclusions that may be evident above the inherent noise of damping measurements. As expected, the control systems of these flight vehicles are altering what the structural dynamics modelers and analysts may be expecting to see. Therefore, it is an important aspect of such studies to understand the interactions. Reference [30] also discussed a dataset that is being generated to specifically assess the effects of control system and structural effects. The data is based on the Ares 1-X flight test. Ares 1-X was a full scale flight test vehicle for the Ares 1 vehicle that flew in 2009. The 327 foot tall vehicle consisted of a four segment Space Shuttle Solid Rocket Booster (SRB) configured as the first and only active stage. A mass simulator of a fifth segment, an upper stage, and the Orion vehicle were also present on the vehicle. This dataset includes both model and flight test data from various phases of the mission (roll-out, pre-launch, lift-off, and ascent are available in general). The model data includes data with and without the control system active and with and without wind disturbances. The intent is to assess the control system effects and to understand the need for tools and data that can help pull out the relevant structural dynamics properties form the launch environment. This reference provided the initial work to assess the lift-off data.

CONCLUSION

The process of identifying modal parameters during launch is a difficult and challenging process. However, there have been a limited number of reported attempts to perform this work that are beginning to show some success. This work has presented a more detailed view of the evolving state of one of these processes. In this work, the launch phase of flight is studied is small moving windows that are intended to keep the variations in modal parameters to a minimum such that stationary assumptions can hold. The typical acceleration data in each window are processed into cross- and autocorrelation functions for each window. Then a semi-automated process using a time-domain system identification tool to extract modal parameters from the correlation functions is used in each window. One of the most challenging aspects of this work is the inherent limited data lengths for the processing windows.

It has been found that this process is hampered by “beating” or “blooms” in the correlation functions that limit the amount of data that can be used in processing. In typical ground based
applications, these effects are rarely seen as frequency-domain averaging is highly effective at eliminating the issue. An alternative approach has been reported that may be applicable in launch processing is the use of multiple iterations of correlation processing using the same base data. This mitigation has been hampered by the lack of a convergence metric to allow the process to terminate.

This work provides a first significant step in providing such a convergence metric. A Hilbert transform is performed on the resulting correlation function and used to generate an analytic representation of the data. The envelope of the correlation function can thus be extracted from the analytic function. The natural log of this envelope is then used to set the data into a format where a linear slope can be estimated. A significant finding is that for a limited class of data (single dominant mode) these slope parameter estimates converge and (for known analytical data) converge to the proper damping ratio of the input mode. A relatively limited but real-world field data set has been used to verify the performance of the simple analytical data studies. The availability of such a convergence process is now allowing more in-depth studies of the entire process.

Several limitations in the current process have been identified. The previously mentioned single dynamic phenomena are an issue. Limited studies have been performed to-date with multiple sensors. The limits of stationarity are not known. Additional studies are needed to understand various secondary features in the process. However, the most significant limitation with this iteration of the process is the deterministic trend to have damping increase with short time records. There are several suggestions for understanding and mitigating this issue which will form the basis of the next round of development studies.

The increasing attempt to obtain high-fidelity identification capability for the launch environment is intended to open the possibility for understanding the launch phase dynamics and justifying the budget, data, and effort to do so. One of these efforts that has garnered recent interest is in the ability to quantify and understand the effects of the control system. The work discussed herein is feeding directly into a series of tasks to specifically study control effects.

APPENDIX A: THEORETICAL BASIS FOR AN OMA PROCESS, NExT

A critical step in the development of NExT was to find a function that could be measured from operational data, but possessed a clear relationship with and a dependence on the modal parameters of the structure. For NExT, the function selected was cross-correlation functions between responses without a measurement of the input force. This section outlines the development of the relationship between the cross-correlation function and modal parameters. The full details of this development can be found in multiple references.

The derivation begins by assuming the standard matrix equations of motion:

\[
[M] \ddot{x}(t) + [C] \dot{x}(t) + [K] x(t) = \{f(t)\}
\]

where

- [M] is the mass matrix;
- [C] is the damping matrix;
- [K] is the stiffness matrix;
- \{f\} is a vector of random forcing functions;
- \{x\} is the vector of random displacements; and t is time.
Equation (1) can be expressed in modal coordinates using the standard modal transformation and diagonalized matrices (assuming proportional damping). A solution to the resulting scalar modal equations can be performed via the convolution or a Duhamel integral and assuming a general forcing function \( \{ f \} \) with zero initial conditions. The solution can be converted back into physical coordinates and specialized for a single input force and a single output using appropriate mode shape matrix entries. The following equation results:

\[
x_{ik}(t) = \sum_{r=1}^{n} \psi_{ir} \psi_{kr} \cdot \int_{-\infty}^{t} f_k(\tau) g^r(t - \tau) d\tau
\]

(2)

where

\[
g^r(t) = \begin{cases} 
0, & \text{for } t < 0; \\
\frac{1}{m_r \omega_n^r} \exp\left(-\zeta_r \omega_n^r t\right) \sin\left(\omega_n^r t\right), & \text{for } t \geq 0;
\end{cases}
\]

\[
\omega_n^r = \omega_n^r \left(1 - \zeta_r^2\right)^{1/2}
\]

is the damped modal frequency;

\[
\omega_n^r \text{ is the } r^{th} \text{ modal frequency;}
\]

\[
\zeta_r \text{ is the } r^{th} \text{ modal damping ratio;}
\]

\[
m_r \text{ is the } r^{th} \text{ modal mass;}
\]

\[
n \text{ is the number of modes;}
\]

\[
\psi_{ir} \text{ is the } i^{th} \text{ component of mode shape } r; \text{ and}
\]

\[
t \text{ is the time.}
\]

The next step of the theoretical development is to form the cross-correlation function of two responses (\( x_{ik} \) and \( x_{jk} \)) due to a white-noise input at a particular input point \( k \). The cross-correlation function \( R \) as the expected value of the product of two responses evaluated at a time separation of \( T \),

\[
R_{ijk}(T) = E[x_{ik}(t + T) x_{jk}(t)]
\]

(3)

where \( E \) is the expectation operator.

Substituting Equation (2) into (3) and recognizing that the force \( f \) is the only random variable, then the expectation operator functions only on the forcing function. Using the definition of the autocorrelation function \( 17 \), and assuming for simplicity that the forcing function is white noise (this is only approximately true), then the expectation operation collapses to a scalar times a Dirac delta function. The Dirac delta function collapses one of the Duhamel integrations embedded in the cross-correlation function. The resulting equation can be simplified via a change of variable of integration \( (\lambda = t - \tau) \). Using the definition of \( g \) from Equation (2) and the trigonometric identity for the sine of a sum results in all the terms involving \( T \) being separated from those involving \( \lambda \). This separation allows terms that depend on \( T \) to be factored out of the remaining integral and out of one of the modal summations. This results in:
\[ \Re_{ijk}(T) = \sum_{r=1}^{n} \left[ A_{ijk}^r \exp \left( -\zeta \omega_n^T \right) \cos \left( \omega_d^T \right) + B_{ijk}^r \exp \left( -\zeta \omega_n^T \right) \sin \left( \omega_d^T \right) \right] \] (4)

where \( A_{ijk}^r \) and \( B_{ijk}^r \) are independent of \( T \), are functions of only the modal parameters, contain completely the remaining modal summation, and are shown below.

\[
\begin{cases}
A_{ijk}^r = \sum_{s=1}^{n} \frac{\alpha_k \psi_{js} \psi_{ks} \psi_{s}}{m^r \omega_d^r m^s \omega_d^s} \int_{-\infty}^{\infty} \exp \left( -\zeta_n^r \omega_n^r - \zeta_s^s \omega_n^s \right) \lambda \cdot \sin \left( \omega_d^r \lambda \right) \left[ \sin(\omega_d^r \lambda) \right] d\lambda.
\end{cases}
\] (5)

Equation (4) is now the key result of this derivation because many time-domain modal analysis algorithms utilized impulse response functions as the input data for estimating the modal parameters. We see that Equation (4) shows that the cross-correlation function has the same characteristics as the impulse response function, a sum of decaying sinusoids with the same damping and frequency as the impulse response function. Thus, cross-correlation functions can be used in place of impulse response functions in these time-domain modal parameter estimation algorithms. Consequently, Equation (4) provides us with the desired function that can be measured from operational data and used to extract the modal parameters. The reader can refer to the provided references for more details of the intermediate steps in this derivation.\textsuperscript{1,4,15} These references also provide verification of this derivation, using the maximum of an autocorrelation function and a cross-correlation identity.

REFERENCES


