Gyroscopic Stabilization of Unstable Dynamical Systems

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Abstract

The use of gyroscopically-induced moments for stabilization has been known about for over a century; Louis Brennan, in 1905, submitted a patent describing the use of gyroscopes to enable a monorail train or other two-wheeled vehicle to operate in a stable manner. Since then, gyroscopes have been adapted to stabilizing and actuating monorails, large oceangoing ships, as well as spacecraft and satellites. However, despite its prolific use in large-scale vehicles, very little has been done to realize gyroscopic stabilization for smaller applications such as robotics. Drawing on previous work applied to monorail-type vehicles, the principles of gyroscopic stability are applied to a similar small vehicle to show the feasibility of the technology on the smaller scale. Through dynamical analysis and simulation, stability and controllability of the system are predicted. Finally, through a prototype, the predictions and analysis are compared to actual system response. Thus, gyroscopic stabilization of small-scale vehicles is proven feasible. The implication of this work is to extend the use of gyroscopes to stabilize more complex dynamical systems, such as bipedal and humanoid robots.

1 Introduction

Gyroscopic stabilization has been around for a very long time. Louis Brennan, in 1905, filed a patent for 'Means of Imparting Stability to Unstable Bodies.' [2] In this patent, he describes the use of a rotating mass that enables a monorail-type train to operate without falling over. The benefits of such a monorail train include less vehicle weight, increased ability to go around turns and up inclines, and not depending on a country’s standard rail spacing (which happens to remain an issue today).

Modern use of gyroscopes for stability and actuation is mostly limited to aerospace, although oceangoing ships also utilize the technology. Even within aerospace, gyroscopic stabilization is further limited mostly to spacecraft and satellites, which use the gyroscopic forces for attitude adjustment and pointing maneuvers. Very little has been done to port the technology to earth-bound uses. The topic has been considered, as in the work by Spry and Girard. [3] Also, an team of undergraduate students implemented a hardware prototype of such a gyroscopically-controlled vehicle for a Capstone Design project, although ultimately the project was unsuccessful in stabilizing the vehicle. [1] Thus, it is the purpose of this study to both investigate the possibility of and test the application of using a Control Moment Gyroscope, or CMG, to stabilize a two-wheeled vehicle much like a skateboard with only two wheels.

The ultimate aim of this project is to further make use of CMG stabilization for other robotic applications, including a CMG-powered two-wheel skateboard and applications to bipedal robots. With robotics becoming ever more prevalent in manufacturing, commercial industries, and even in people’s personal lives, the issue of mobility is an increasing area of interest. Robotics today are less mobile than the inspiration for their designs, especially with humanoid
robots; the human body is made up of hundreds of muscles and connections that allow for precise feedback control of our bodily positions, and all of this is done subconsciously. Robots, on the other hand, have limited amounts of actuation, and what is available must be used for both stabilization as well as movement. Robots are not over-actuated like humans are. Thus, utilizing some secondary method of stabilization will be key to the future of robotics, and the use of CMGs for such an application is a low-power, high-torque solution to a complicated problem.

2 Methodology

To begin, the skateboard-like system was modeled as an inverted-pendulum problem. This simplified the first rendition of modeling of the system, while still allowing for sufficient system dynamics to be considered. The inverted-pendulum was chosen because the first prototype would only be able to rotate and fall over; it would not translate at all. With the system thus described, the first step was to derive its equations of motion.

2.1 Definitions

First, the coordinate frames were defined: (see Figures 1 and 2 for coordinate definitions)

- A-Frame: the earth-fixed inertial reference frame
- B-Frame: fixed to the vehicle body
- C-Frame: fixed to the CMG casing
- G-Frame: fixed to the CMG wheel

![Figure 1: Cart and Frames from Back](image)
Figure 2: Cart and Frames from Side

Once the frames were defined, other properties of the vehicle were defined as follows:

- $\phi$ = Roll angle of the vehicle B-Frame about the 1-axis with respect to the A-Frame (rad)
- $\alpha$ = Gimbal angle of the CMG C-Frame about the 2-axis with respect to the B-Frame (rad)
- $m_B$ = Mass of the vehicle, including everything except the CMG assembly (kg)
- $m_{C+G}$ = Mass of the CMG assembly (kg)
- $d_1$ = Distance from ground to the center of mass of the vehicle body (m)
- $d_2$ = Distance from ground to the center of mass of the CMG assembly (m)
- $I_B$ = Mass Moment of Inertia tensor for the vehicle body, including everything except the CMG assembly (kg*m^2). The notation used includes subscripts to indicate the axis of inertia, and the products of inertia are disregarded since they are relatively small compared to the principle axes of inertia: thus, about the vehicle-body roll axis, $I = I_{B11}$.
- $I_G$ = Mass Moment of Inertia tensor for the CMG assembly (kg*m^2)
- $\Omega$ = Rotation speed of the CMG wheel (rad/s)

Notation used: the generalized method of referring to a vector is to underline it, like so $x$. A unit directional vector is defined both by its frame of reference as well as a hat: $\hat{b}_2$. Further, to denote a relative vector from the C-Frame in
the B-Frame, it is written: $\omega_B$. Finally, the abbreviations for sin and cosine are $s$ and $c$ respectively: $\sin(x) = s(x)$.

2.2 Equations of Motion

In order to accurately model the system’s behavior, a differential equation of motion describing the system is necessary. The LaGrange method is a way to derive the equations of motion if the system’s geometry and other physical characteristics are known, as well as information about how the system will move. Thus, first were needed the equations defining the system’s kinetic and potential energies, which depend on the system’s physical properties and motion. To begin, the angular velocities of the system were found in the B-Frame relative to the inertial A-Frame. Standardizing the frame of reference for the various physical properties ensures an accurate model, which allows for more accurate simulation of the system:

\[
\begin{align*}
\omega_B^A &= \dot{\phi} \hat{b}_1 \\
\omega_C^A &= \dot{\phi} \hat{b}_1 + \dot{\alpha} \hat{b}_2 \\
\omega_G^A &= (\dot{\phi} + \Omega s(\alpha)) \hat{b}_1 + \dot{\alpha} \hat{b}_2 + \Omega c(\alpha) \hat{b}_3
\end{align*}
\]

Since the system being considered will not include any translational motion (it is essentially like a cradle, with the CMG on top and the ability to rock back and forth), the angular velocity terms above are the only velocity terms that will apply to the energy equations. Next, the kinetic energies are:

\[
\begin{align*}
T_B &= \frac{1}{2} [c_1 \dot{\phi}^2] \\
T_C &= \frac{1}{2} [c_2 \dot{\phi}^2 + c_3 \dot{\alpha}^2] \\
T_G &= \frac{1}{2} [c_4(\dot{\phi} + \Omega s(\alpha))^2 + c_5(\Omega c(\alpha)(\dot{\phi} + \Omega s(\alpha))) + c_6 \dot{\alpha}^2 + c_7(\Omega c(\alpha))^2]
\end{align*}
\]

where:

\[
\begin{align*}
c_1 &= I_{B11} \\
c_2 &= c^2(\alpha)I_{C11} + s^2(\alpha)I_{C33} \\
c_3 &= I_{C22} \\
c_4 &= c^2(\alpha)I_{G11} + s^2(\alpha)I_{G33} \\
c_5 &= c(\alpha)s(\alpha)I_{G11} - c(\alpha)s(\alpha)I_{G33} \\
c_6 &= I_{G22}
\end{align*}
\]

That leaves us with the equation for kinetic energy:

\[T = T_B + T_C + T_G\]
Then, the potential energies of the system are defined as:

\[ V_B = gd_1m_Bc(\phi) \] (8)

\[ V_{C+G} = gd_2m_{C+G}c(\phi) \] (9)

Finally, the LaGrange method, which is used to calculate the differential equations of motion, is defined as the following, with \( L = \sum T - \sum V \):

\[
\frac{d}{dt} \left( \frac{\delta L}{\delta \dot{q}_i} \right) - \frac{\delta L}{\delta q_i} = Q_i
\]

Since the control method for this system is to send a desired gimbal-rate to the CMG’s gimbal motor, the only equation of motion that really matters is the one describing the motion of the vehicle roll angle, or the differential equation in the variable \( \phi \). The equation of motion from the LaGrange method is as follows:

\[
0 = \ddot{\phi} \left[ I_{B11} + I_{C11}c^2(\alpha) + I_{G11}c^2(\alpha) \right] \\
+ \ddot{\alpha} \left[ I_{C33} + I_{G33} \right] \\
+ \ddot{\phi} \left[ m_Bd_1^2 + 2m_{C+G}d_2^2 \right] \\
+ \dot{\alpha} \Omega \left[ -4I_{G11}c(\alpha) + 5I_{G33}c(\alpha) + 6I_{G11}c^3(\alpha) - 6I_{G33}c^3(\alpha) \right] \\
+ gsc(\phi) \left[ -d_1m_B - d_2m_{C+G} \right] \\
+ \dot{\alpha}\dot{\phi}s(2\alpha) \left[ -I_{C11} - I_{C33} - I_{G11} + I_{G33} \right]
\]

Simplified about the equilibrium position of \( \phi = \alpha = 0 \), the equation becomes:

\[
\ddot{\phi} = \frac{b}{a} \dot{\alpha}
\] (11)

where:

\[
a = I_{B11} + I_{C11} + I_{G11} + m_Bd_1^2 + 2m_{C+G}d_2^2 \\
b = \Omega(I_{G33} - 2I_{G11})
\]

Thus, as is to be expected, the angular acceleration of the roll axis of the vehicle is largely dependent on the angular momentum of the CMG wheel as well as the physical properties of the vehicle itself.

### 2.3 Simulation

Once the equation describing the system was obtained, the model was then tested by means of computer simulation. First, the system was described in state-space form \( \dot{x} = Ax + Bu \):

\[
\dot{x} = \begin{bmatrix} \dot{\phi} \\ \dot{\alpha} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ b/a \end{bmatrix} \dot{\alpha}
\] (12)
First, it is important to note that since the control input is the gimbal angular rate, or \( \dot{\alpha} \), here it shows up as the control input \( u \). Now, the control input is calculated based on a set of gains multiplied by the state values, and then divided by the angular momentum of the CMG wheel:

\[
\dot{\alpha} = \frac{[K][z]}{I_{G33}\Omega}
\] (13)

This equation results in a scalar value, which is multiplied by the \( B \) matrix as described above to actuate the system. This is a standard proportional-derivative control method, since the control input is calculated based on both the angular positions of the CMG and the vehicle, as well as the angular rate of the vehicle. This is called PD control, and is one of the most commonly used methods to control systems in order to achieve a desired output. In this case, the desired output is a vehicle and gimbal angle of zero. Figures 4 and 5 depict simulation results for an initial vehicle angle of -15 degrees. The simulations were run at a discretized rate of 200Hz and integrated with the Runge-Kutta 4th-order approximation. Note how the system nearly achieves equilibrium within about 2 seconds. This is entirely tunable with the feedback gains, which, when changed, can bring the system to equilibrium faster but with more oscillation, or more slowly with fewer oscillations.

2.4 Hardware Implementation

Once the simulations have been done to find the optimal desired system response via tuning of the gain values, the controller is implemented in hardware. A simple vehicle was built using rapid-prototyping techniques, and the CMG, motor controllers, and Inertial Motion Sensor were installed and configured.\(^1\) See Figure 3 which depicts the system:

3 Results

The following Figures 4 through 6 depict the simulated system responses for a particular set of gains, as well as an average of three sets of test data from the hardware. These data were obtained utilizing the same gains in simulation as in hardware implementation and the same initial conditions.

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\(^1\) A caveat: the CMG units were purchased without available documentation other than for the motors on the units, and thus a lot of in-house testing was done to verify the abilities of the CMGs. For the simulations as well as hardware tests, the maximum CMG wheel speed was used, at 2900 RPM. Additionally, since this system was to be the first in a series of project builds and to serve as a method of proof-of-concept and hardware, all power and communication was sent through umbilical cables that were attached to microcontrollers and the PC running the control software.
4 Discussion

4.1 Results

The prototype vehicle was tested many times with various controllers and for various initial conditions, and the physical behavior of the system was very close to the expected behavior found through simulation. As can be seen in Figures 4 and 5, the vehicle roll angle and vehicle angular roll rate have very distinct sections that match trajectories with the simulations. However, the data do not match exactly, and it is believed that is due to a very imprecise gimbal motor encoder.\textsuperscript{2} Due to the fluctuations in gimbal angle readings, the system was being actuated on inaccurate gimbal data. However, the controller was robust enough to handle such imperfections in the system, and therefore the testing was concluded as successful since in all cases the vehicle was brought to a state of equilibrium.\textsuperscript{3}

\textsuperscript{2}The gimbal motor is a simple DC servomotor with output through a gearbox. Although the gearbox increases the available torque used to turn the CMG, it has drawbacks as well: first, the motor and gearbox have an inherent backlash, which allows the CMG to be tilted $\pm 5^\circ$. This shows up in the hardware tests as an inability to switch the gimbal rate direction instantaneously, affecting the actuating power of the CMG. In addition to the backlash, through many simulations it was found that, although the gimbal rate command was at or very near zero, the gimbal angle reading fluctuated $\pm 10^\circ$ continuously.

\textsuperscript{3}The states of equilibrium of the vehicle were designed to be as close to a vehicle angle of zero, although the addition of the umbilical cables made it difficult to predict where such equilibrium states would occur. Thus, when the system would reach equilibrium, the vehicle roll angle was not exactly zero, but $\pm 2^\circ$ or so.
Figure 4: Vehicle Roll Angle Data Comparison
Figure 5: Vehicle Roll Rate Data Comparison
Figure 6: Gimbal Angle Data Comparison
4.2 Experimental Revisions

If this process were to be repeated, a couple things would be changed in order to gather more accurate data. First, the CMG itself would be designed and built in such a way as to prevent the inconsistencies that were seen in the gimbal angular position due to the backlash in the gearbox and CMG housing design. The CMG motor would also be changed to allow for more precise control and measurement; a stepping motor would potentially serve as a viable solution. Finally, although the simulation code was adaptable, and the complete equations of motion were known, the equations used in the simulations were linearized about the desired equilibrium state of zero. Thus, a more general case utilizing the full equations of motion may lead to further insight into the system’s response to various initial conditions.

4.3 Future Work

With such unpredictable system responses due to imprecise hardware, the test data were individually inconclusive. However, when the test data were averaged, recognizable trends that follow the predicted trajectories occurred, which led to an overall conclusion of success of the model. Thus, with a system of modeling that works, as well as an integrated CMG control scheme that results in stability of a vehicle, the next step is to expand the skateboard-like vehicle design into an autonomous CMG-powered vehicle that uses two CMGs to develop the forces necessary for movement. Beyond that, the knowledge and experience gained throughout this experiment will prove useful for implementing CMG stabilization in other types of robotics.

5 Conclusion

The process described above follows the path of generally accepted control design; the system is identified, described mathematically, and modeled with differential equations of motion, the controller is designed and the gains are tuned, and then the model and control response are compared to the system response to obtain information about the accuracy of the model. With this specific system, the challenges that were inherent were mainly caused by inaccurate gimbal angular position data and the backlash in the gearbox of the gimbal motor. However, although these posed problems for the vehicle’s stability and response, the model did accurately predict the trajectory the system would follow from various initial conditions, and thus the model was deemed sufficiently accurate.

Thus, with a successful model and a better understanding of the hardware that is available, further work will be pursued in this field of interest. The CMG itself may need to be fixed before it can be applied to more difficult control situations, and with more complicated problems come more complicated dynamics, but the initial experiment has proven the CMG’s usefulness in land-based robotics, and the groundwork has been laid for the path that leads to more stable robotics in the years ahead.
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References

