Current Status of Orbit Determination methods in PMO

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Abstract

Satellite orbit determination (OD) methods have evolved a lot over the past 50 years in Purple Mountain Observatory. This article provides an overview of OD methods in PMO. Because of the sophistication and complexity encompassing OD, we only introduced the force models we considered for each OD method, technical details about OD will not be talked about in depth.

1. Introduction

Satellite orbit determination (OD) can be described as the method of determining the position and velocity of an orbiting object, such as an interplanetary spacecraft or an Earth-orbiting satellite, using the set of measurements collected onboard the satellite or by ground-based tracking stations.

OD methods can be classified into 3 categories:

1) Pure analytical method, which considered General perturbation (GP)
2) Semi-analytical method
3) Precision OD (POD), which considered special perturbation (SP)–numerical integration

2. Pure analytical method

The Force models we considered for the pure analytical method is as follows:

1. Geopotential - low-order zonal terms

\[ a_i(t-t_0) = 0 \]
\[ e_i(t-t_0) = 0 \]
\[ i_i(t-t_0) = 0 \]
\( \Omega_i(t-t_0) = -\frac{A_2}{p^2} m \cos i(t-t_0) \)

\( \omega_i(t-t_0) = \frac{A_2}{p^2} n \left( 2 - \frac{5}{2} \sin^2 i \right) (t-t_0) \)

\( M_i(t-t_0) = \frac{A_2}{p^2} n \left( 1 - \frac{3}{2} \sin^2 i \right) \sqrt{1-e^2} (t-t_0) \)

where \( n = a^{-3/2}, p = a \left( 1 - e^2 \right) \)

2) Long period perturbation

\( a_i^{(1)}(t) = 0, \)

\( e_i^{(1)}(t) = -\left( \frac{1-e^2}{e} \right) t_i^{(1)}(t), \)

\( t_i^{(1)}(t) = -\frac{1}{12} \frac{A_1}{p^2} \sin 2 \cos 2f \cos 2\omega - \frac{A_2}{p^2} \frac{\sin 2i}{(4-5 \sin^2 i)} \left( \frac{7}{24} - \frac{5}{16} \sin^2 i \right) e^2 \cos 2\omega \)

\[ - \frac{3}{4p} \left( \frac{A_1}{A_2} \right) \cos i \cdot e \sin \omega + \frac{1}{p^2} \left( \frac{A_1}{A_2} \right) \frac{\sin 2i}{(4-5 \sin^2 i)} \times \left( \frac{9}{28} - \frac{3}{8} \sin^2 i \right) e^2 \cos 2\omega \]

\( \Omega_i^{(1)}(t) = -\frac{1}{6} \frac{A_2}{p^2} \cos i \cos 2f \sin 2\omega - \frac{A_2}{p^2} \frac{\cos i}{(4-5 \sin^2 i)} \times \left( \frac{7}{3} - 5 \sin^2 i + \frac{25}{8} \sin^4 i \right) e^2 \sin 2\omega \)

\[ + \frac{3}{4p} \left( \frac{A_1}{A_2} \right) \times \text{ctgi} \cdot e \cos \omega + \frac{1}{p^2} \left( \frac{A_1}{A_2} \right) \frac{\cos i}{(4-5 \sin^2 i)} \times \left( \frac{18}{7} - 6 \sin^2 i + \frac{15}{4} \sin^4 i \right) e^2 \sin 2\omega, \]

\( \omega_i^{(1)}(t) = \frac{A_2}{p^2} \left[ \sin^2 i \left( \frac{1}{8} + \frac{1-e^2}{6e^2} \cos 2f \right) + \frac{1}{6} \cos^2 i \cos 2f \right] \times \sin 2\omega - \frac{A_2}{p^2} \frac{1}{(4-5 \sin^2 i)} \left[ \sin^2 i \left( \frac{25}{3} - \frac{245}{12} \sin^2 i \right) \right. \]

\[ + \frac{25}{2} \sin^4 i \right] e^2 \left( \frac{7}{3} - \frac{17}{2} \sin^2 i + \frac{25}{6} \sin^4 i \right) \]

\[ - \frac{75}{16} \sin^6 i \right] \sin 2\omega + \frac{3}{4p} \left( \frac{A_1}{A_2} \right) \frac{1}{e \sin i} \left[ (1+e^2) \sin^2 i \right. \]

\[ - \frac{1}{2p} \left( \frac{A_1}{A_2} \right) \frac{1}{(4-5 \sin^2 i)} \left[ \sin^2 i \left( \frac{18}{7} \right. \right. \]

\[ - \frac{87}{14} \sin^2 i + \frac{15}{4} \sin^4 i \right] e^2 \left( \frac{18}{7} - \frac{69}{7} \sin^2 i \right) \]

\[ + \frac{90}{7} \sin^4 i - \frac{45}{8} \sin^6 i \right] \sin 2\omega, \]
\[ M_i^{(1)}(t) = -\frac{A_e}{p^2} \sqrt{1-e^2} \sin^2 i \left( \frac{1}{8} + \frac{1+e^2/2}{6e^2} \cos 2f \right) \sin 2\omega + \frac{A_e}{p^2} \sqrt{1-e^2} \sin^2 i \left[ \frac{25}{12} - \frac{5}{2} \sin^2 i \right] - e^2 \left( \frac{7}{12} - \frac{5}{8} \sin^2 i \right) \sin 2\omega. \]

3) Short period perturbation

According to the limitation of the page, we only write the expression of \( a_i^{(1)}(t), e_i^{(1)}(t), i_i^{(1)}(t) \) here.

\[ a_i^{(1)}(t) = \frac{A_e}{a} \left[ 2 - \frac{1}{2} \sin^2 i \left[ \left( \frac{a}{r} \right)^3 - \left( 1-e^2 \right)^{-3/2} \right] \right] \sin^2 i \left( \frac{a}{r} \right)^3 \cos 2(f + \omega), \]

\[ e_i^{(1)}(t) = \frac{(1-e^2)}{e} \left[ \left( \frac{1}{2a} \right) a_i^{(1)}(t) - t g i \cdot i_i^{(1)}(t) \right], \]

\[ i_i^{(1)}(t) = \frac{A_e}{4p^2} \sin 2i \left[ e \cos(f + 2\omega) + \cos 2(f + \omega) + \frac{e}{3} \cos(3f + 2\omega) \right]. \]

2. Atmospheric drag

In the pure analytical method, the atmospheric drag on mean motion is considered as linear in time.

The accuracy of the pure analytical method is kilometers. It was first developed in the 1960s and mainly used for theoretical studies or satellite orbit design. The speed of the pure analytical method is fast, but not too accurate.

3. Semi-analytical method

The equation of motion of the satellite in the semi-analytical method can be described as:

\[ \frac{d\sigma}{dt} = \delta \times n + \left( \frac{d\sigma}{dt} \right)_n + \left( \frac{d\sigma}{dt} \right)_D + \left( \frac{d\sigma}{dt} \right)_R + \left( \frac{d\sigma}{dt} \right)_{SM} \]

Where \( n \) expresses the mean motion, while \( \left( \frac{d\sigma}{dt} \right)_n, \left( \frac{d\sigma}{dt} \right)_D, \left( \frac{d\sigma}{dt} \right)_R, \left( \frac{d\sigma}{dt} \right)_{SM} \), respectively, express the perturbations of Earth zonal term, air drag, solar radiation pressure and sun-moon gravitation.

The force models we considered for the semi-analytical method are as follows:
The accuracy of the semi-analytical method is hundred meters. It is mainly used for research on atmosphere model and space catalog. The model was first developed in 1977. It maintains a balance between speed and accuracy.

4. Numerical method

The equation of motion of the satellite in the numerical method is:

\[ \ddot{r} = \vec{F}_0 + \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 \]

where the \( \vec{F}_0, \vec{F}_1, \vec{F}_2, \vec{F}_3, \vec{F}_4 \) respectively express the perturbation of center-gravitation, Earth zonal term, Sun-Moon gravitation, air drag and solar radiation pressure.

The force models we considered for the precision orbit determination are as follows:

1) Geopotential – JGM3 50×50

We can write the Earth’s gravity potential in the form:

\[ U_{xs} = \frac{GM_e}{r} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \left( \frac{a_{lm}}{r^l} \right) \vec{P}_{lm} (\sin \phi \left[ \vec{C}_{lm} \cos(m\lambda) + \vec{S}_{lm} \sin(m\lambda) \right] \]

where the coefficients refer to the JGM3 model.

2) Sun and Moon – JPL DE200

According to Newton’s law of gravity, the perturbation of Sun and Moon can be calculated by the formula:

\[ \vec{F}_2 = \sum_{j=1}^{2} (-GM_j) \left( \frac{\vec{R}_j}{R_j^3} + \frac{\vec{\Delta}_j}{\Delta_j^3} \right) \]

where the position of the Sun and Moon can be obtained from the JPL DE200.

3) Atmospheric drag - DTM94 / MSIS 1990

The acceleration of atmosphere drag can be written in the form:

\[ \vec{F}_3 = -\frac{1}{2} \rho C_D \frac{A}{m} V_r \vec{V}_r \]

In our program, the air density are computed by the DTM94 and MSIS1990. According to the
practice, both models are available.

4) Solar radiation pressure

For the satellites with a complicated shape, its effective area can not be calculated easily. The Box-Wing model was involved in our software to solve the problem.

\[
\vec{F}_d = -P_d \frac{A}{m} \alpha \cos \theta \left[ 2 \left( \frac{\delta}{3} + \rho \cos \theta \right) \hat{n} + (1 - \rho) \hat{s} \right]
\]

5) Solid and ocean tidal perturbation

The formula of computing the solid perturbation is:

\[
\begin{aligned}
\Delta C_{lm} &= \frac{(-1)^m}{a_e \sqrt{4\pi(2-\delta_{\text{bm}})}} \sum_k k^l H_k \cdot \begin{cases} 
\cos \Theta_k, (l - m = \text{even}) \\
\sin \Theta_k, (l - m = \text{odd}) 
\end{cases} \\
\Delta S_{lm} &= \frac{(-1)^m}{a_e \sqrt{4\pi(2-\delta_{\text{bm}})}} \sum_k k^l H_k \cdot \begin{cases} 
(-\sin \Theta_k), (l - m = \text{even}) \\
(+\cos \Theta_k), (l - m = \text{odd}) 
\end{cases}
\end{aligned}
\]

The formula of computing the ocean tidal perturbation is:

\[
\begin{aligned}
\left( \Delta C_{lm} \right)_{\text{OT}} &= F_{lm} \sum_k A_{km} \\
\left( \Delta S_{lm} \right)_{\text{OT}} &= F_{lm} \sum_k B_{km}
\end{aligned}
\]

\[
F_{lm} = \frac{4\pi a_e^2 \rho_{\text{os}}}{100M_e} \left( \frac{l + m}{(l - m)!(2l + 1)(2 - \delta_{\text{bm}})} \right)^{1/2} \left( \frac{1 + k_i}{2l + 1} \right)
\]

\[
\begin{pmatrix} A_{k l m} \\ B_{k l m} \end{pmatrix} = \begin{pmatrix} C_{k l m}^+ + C_{k l m}^- \\ S_{k l m}^+ - S_{k l m}^- \end{pmatrix} \cos \Theta_k + \begin{pmatrix} S_{k l m}^+ - S_{k l m}^- \\ C_{k l m}^- - C_{k l m}^+ \end{pmatrix} \sin \Theta_k
\]

6) Relativistic effects

For high precision requirement, the relativistic correction was involved in the program.

\[
\vec{A}_{\text{REL}} = \frac{GM_e}{c^2 r^3} \left[ 2(\beta + \gamma) \frac{GM_e}{r} - \gamma \left( \vec{r} \cdot \hat{r} \right) \vec{r} + 2(1 + \gamma) \left( \vec{r} \cdot \hat{r} \right) \hat{r} \right]
\]

7) Earth radiation pressure

The Earth radiation pressure can be computed with the formula:

\[
\vec{A}_{\text{earth}} = (1 + \eta_e) \frac{A}{m} \sum_{j=1}^{N} \frac{A_j}{\alpha_j^2} \left[ (\tau \cdot aE_s \cos \theta_s + eM_b) \hat{r} \right]
\]

As for the numerical integration, we use Adams-Cowell multi-step methods, while using RKF 8 for beginning. The magnitude of the accuracy is 10 centimeters.
It is used when high accuracy is required, such as the processing of SLR data. Precise orbit determination software has been developed by us. It is not too fast, but has a high accuracy.

The POD software was developed and used since 1990s and is still improving and developing today.

5. Summary

In this article, three categories of orbit determination methods in PMO are introduced. We choose different methods according to different purposes and accuracy demands. All software mentioned above are developed by PMO independently, and are still developing today.