Analysis on Orbit Characteristics of a Space Object Based on a Low Thrust Orbit Transfer

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Abstract: During the observation of the space object, once the space object surveillance system creates an orbit elements set, it will rapidly correlate the whole trajectory, calculated according to the observed data, with the orbit elements set in order to complete the matching of the space object. When collision or orbit maneuver events happen, the orbit of the spacecraft will change, and its amplitude will closely relate to the size of debris and the relative state of two objects. This paper compares the effect of the perturbation force and that of a low thrust orbit transfer on the orbit, and accomplishes the orbit maneuver assessment and orbit characteristics analysis based on basic orbit dynamics simulation, which provides the foundation for the correlation of space objects.

Key words: low thrust, orbit dynamics simulation, orbit matching

1 Introduction

Generally, the space object regularly moves on a certain orbit, its orbital elements are the main parameters to represent its motion law, including the altitude, shape and motion characteristic of the orbit. According to the altitude, orbits can be classified as LEO, MEO and HEO; according to the shape, orbits can be classified as near-circular orbit, small elliptic orbit, and large elliptic orbit; according to the motion characteristic, orbits can be classified as Sun synchronous orbit, geosynchronous orbit, and geostationary orbit. If the influence of each perturbation force is neglected, the space object shall move along the inherent orbit, its six orbital elements are the main parameters to determine its motion law, and also the foundation for space object identification. Thus, space object identification can be realized through matching observation data with the orbital elements of known objects. However, the space object is not only influenced by the universal gravitation, but also influenced unavoidably by the disturbance of other forces (such as the departure of terrestrial universal gravitation from the central gravitational field, remainder atmosphere drag, the gravitation of the Sun and Moon, the electromagnetic field effect, solar light pressure, etc.); thus, the orbit will finally change. At the same time, the space object, especially the in-orbit spacecraft, will perform orbit adjustment or orbit maneuver according to the requirement of the mission. The orbit will be instantaneously changed a lot through the conventional impulse orbit transfer, which can be analyzed by comparing orbital elements. However, the orbit will be changed slowly through the low thrust orbit maneuver or through the influence of perturbations, so both are easily confused. In order to assess the influence of the perturbations and that of low thrust,
this paper accomplishes the thorough analysis and simulation based on a low thrust maneuver model and a perturbation model, which lays the foundation for the upcoming space object orbit identification.

2. Dynamics model

The selection of an appropriate dynamics model will influence both the computational time and the integral accuracy. Since the orbital elements will continuously and slowly change under the secular effect of low thrust, it is necessary to improve the integral velocity and avoid the singularity of digital integration. This paper applies the generalization modified equatorial orbit elements to avoid the appearance of a singularity during the digital integration. The equations of motion are:

\[
\dot{h} = h \cdot n \cdot f_i,
\]

\[
\dot{f} = h \cdot \sin L \cdot f_i + [h \cdot \cos L + n \cdot (f \cdot L + f)] \cdot f_i - n \cdot X \cdot g \cdot f_n,
\]

\[
\dot{g} = -h \cdot \cos L \cdot f_i + [h \cdot \sin L + n \cdot (g \cdot L + g)] \cdot f_i + n \cdot X \cdot f \cdot f_n,
\]

\[
\dot{h} = \frac{1}{2} \cdot n \cdot s^2 \cdot \cos L \cdot f_n,
\]

\[
\dot{k} = \frac{1}{2} \cdot n \cdot s^2 \cdot \sin L \cdot f_n,
\]

\[
\dot{L} = n \cdot X \cdot f_n + \frac{1}{n^2 \cdot h \cdot \mu}
\]

where

\[
n = \frac{h}{1 + f \cdot \cos L + g \cdot \sin L}, \quad X = h \sin L - k \cos L, \quad s^2 = 1 + h^2 + k^2
\]

According to the above equations, the modified equatorial orbital elements can be converted into Cartesian coordinates \((\vec{r}, \vec{v})\), shown as below:

\[
r_x = \frac{r}{s^2} [\cos L + (h^2 - k^2) \cos L + 2hk \sin L]
\]

\[
r_y = \frac{r}{s^2} [\sin L - (h^2 - k^2) \sin L + 2hk \cos L]
\]

\[
r_z = \frac{2r}{s^2} (h \sin L - k \cos L)
\]

\[
v_x = -\frac{1}{hs^2} [\sin L + (h^2 - k^2) \sin L - 2hk \cos L + g - 2fhk + (h^2 - k^2) g]
\]

\[
v_y = -\frac{1}{hs^2} [-\cos L + (h^2 - k^2) \cos L + 2hk \sin L - f + 2ghk + (h^2 - k^2) f]
\]

\[
v_z = \frac{2}{hs^2} [h \cos L + k \sin L + fh + gk]
\]
where
\[ r = \frac{h^2 \mu}{1 + f \cos L + g \sin L} \]

Considering the orbit description of the space object, the modified equatorial orbital elements can be converted into classical orbital elements.
\[
\begin{align*}
    h &= \sqrt{a(1-e^2)/\mu} \quad f = e \cos(\omega + \Omega) \\
    g &= e \sin(\omega + \Omega) \quad h = \tan(i/2) \cos \Omega \\
    k &= \tan(i/2) \sin \Omega \quad L = \omega + \Omega + \theta
\end{align*}
\]

3 Perturbation Analysis

With the variation of the orbit altitude, the magnitude of each perturbation force also changes ceaselessly, thus the influence on the orbits of the spacecraft is also different. The spacecraft in LEO is mainly influenced by the terrestrial non-spherical gravitational perturbation and the atmospheric drag, other perturbation factors are all higher order small quantities. When the orbit altitude is relatively low (less than 600km, for example), the influence of the solar/lunar perturbation and solar light pressure can be neglected; when the orbit altitude increases (more than 800km), the influence of atmospheric drag can be neglected and only the influence of terrestrial non-spherical perturbation can be considered. Generally, only the influence of the \( J_2 \) term perturbation is considered in orbit planning. Tables 3.1, 3.2 and 3.3, respectively, show the influence of the main perturbation force of different orbit altitudes on the orbit prediction accuracy in the time interval of one day, where Y means the corresponding perturbation factors in the second column of the left hand are considered (unit: meter).

Table 3.1: the influence of each perturbation force on the low earth orbit (350km)

<table>
<thead>
<tr>
<th>Perturbation force</th>
<th>Degree of the earth potential function</th>
<th>Atmosphere drag</th>
<th>Solar light pressure</th>
<th>Solar gravitation</th>
<th>Lunar gravitation</th>
<th>Error of one day</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>0</td>
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<tr>
<td></td>
<td>0</td>
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<td>Y</td>
<td>Y</td>
<td>Y</td>
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</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>153</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>967</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.72</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5.23</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>13.3</td>
</tr>
<tr>
<td>Perturbation force</td>
<td>Degree of the earth potential function</td>
<td>Atmospheric drag</td>
<td>Solar light pressure</td>
<td>Solar gravitation</td>
<td>Lunar gravitation</td>
<td>Error of one day</td>
</tr>
<tr>
<td>-------------------</td>
<td>----------------------------------------</td>
<td>-------------------</td>
<td>---------------------</td>
<td>-------------------</td>
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</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Maximum in radial direction</td>
<td>0</td>
<td>271.9</td>
<td>83.</td>
<td>59.6</td>
<td>1.15</td>
<td>0.67</td>
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<tr>
<td>Maximum in tangential direction</td>
<td>0</td>
<td>933.2</td>
<td>288</td>
<td>2226</td>
<td>4.10</td>
<td>2.86</td>
</tr>
<tr>
<td>Maximum in normal direction</td>
<td>0</td>
<td>163.7</td>
<td>73.</td>
<td>1.9</td>
<td>0.44</td>
<td>13.3</td>
</tr>
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</table>

Table 3.2: the influence of each perturbation force on the medium Earth orbit (600 km)

<table>
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<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>Perturbation force</td>
<td>Degree of the earth potential function</td>
<td>Atmospheric drag</td>
<td>Solar light pressure</td>
<td>Solar gravitation</td>
<td>Lunar gravitation</td>
<td>Error of one day</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>50</td>
<td>4</td>
<td>16</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>356.4</td>
<td>142.</td>
<td>158.</td>
<td>16.3</td>
<td>8.48</td>
</tr>
<tr>
<td>Maximum in radial direction</td>
<td>0</td>
<td>219.0</td>
<td>49.6</td>
<td>6.1</td>
<td>10.5</td>
<td>0.53</td>
</tr>
<tr>
<td>Maximum in tangential direction</td>
<td>0</td>
<td>925.6</td>
<td>261.</td>
<td>353.</td>
<td>29.8</td>
<td>1.09</td>
</tr>
<tr>
<td>Maximum in normal direction</td>
<td>0</td>
<td>66.4</td>
<td>58.5</td>
<td>0.1</td>
<td>1.59</td>
<td>14.7</td>
</tr>
</tbody>
</table>

Table 3.3: the influence of each perturbation force on the medium orbit (800 km)
<table>
<thead>
<tr>
<th>Perturbation force</th>
<th>Degree of the earth potential function</th>
<th>50</th>
<th>4</th>
<th>16</th>
<th>50</th>
<th>50</th>
<th>50</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Atmospheric drag</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Solar light pressure</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Solar gravitation</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td></td>
<td>Lunar gravitation</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Error of one day</td>
<td>Position error</td>
<td>0</td>
<td></td>
<td></td>
<td>192.</td>
<td>41.6</td>
<td>115.</td>
<td>8.78</td>
</tr>
<tr>
<td></td>
<td>Maximum in radial direction</td>
<td>0</td>
<td></td>
<td></td>
<td>164.</td>
<td>10.8</td>
<td>5.5</td>
<td>6.30</td>
</tr>
<tr>
<td></td>
<td>Maximum in tangential direction</td>
<td>0</td>
<td></td>
<td></td>
<td>436.</td>
<td>84.2</td>
<td>267.</td>
<td>23.5</td>
</tr>
<tr>
<td></td>
<td>Maximum in normal direction</td>
<td>0</td>
<td></td>
<td></td>
<td>92.7</td>
<td>13.9</td>
<td>0.09</td>
<td>0.70</td>
</tr>
</tbody>
</table>
Fig. 3.1 Analysis on the influence of the low thrust perturbation (0.0083N) of LEO satellites (350km)

Fig. 3.2 Analysis on the influence of the low thrust perturbation (0.0083N) of LEO satellites (600km)
Fig. 3.3 Analysis on the influence of the low thrust perturbation (0.0083N) of LEO satellites (800km)

4 Analysis on the influence of orbit elements

This paper analyzes the influence of the low thrust perturbation on the space position of spacecraft in the previous sections. However, Keplerian orbital elements are usually used in the description of the motion of spacecraft. This paper analyzes the influence of the low thrust on the space object orbit elements in this section.
Fig 4.1: the influence of the low thrust perturbation on the orbital elements (0.083N) of LEO satellites (350km)

Fig 4.2: the influence of the low thrust perturbation on the orbital elements (0.0083N) of LEO satellites (350km)

Fig 4.3: the influence of the low thrust perturbation on the orbital elements
Fig 4.4: the influence of the low thrust perturbation on the orbital elements (0.083N) of LEO satellites (600km)

Fig 4.5: the influence of the low thrust perturbation on the orbital elements (0.0083N) of LEO satellites (350km)
Fig 4.6: the influence of the low thrust perturbation on the orbital elements 
(0.00083N) of LEO satellites (600km)

Fig 4.7: the influence of the low thrust perturbation on the orbit elements 
(0.083N) of LEO satellites (800km)
According to figures shown above, if the low thrust increases by an order of
magnitude, orbital elements will also increase correspondingly by the same order of magnitude. Under the influence of low thrust perturbation, the semi-major axis will also continuously change with the error increasing, which shows that the orbit energy also changes continuously; the right ascension of the ascending node also continuously changes with the error increasing, which shows that the orbital plane also continuously rotates. However, the whole variation will periodically vibrate with the orbital period.

5 Conclusion

During the space object identification, the newly-formed candidate orbit elements set must have a series of final examinations before the review of analysts. If the processing mode is an orbit transfer mode, then the orbit identification algorithm will be used to search for the possible mother set of the orbital element sets. The satellite will move into a new trajectory after a satellite maneuver. If the conventional impulse orbit transfer is used, then the maneuver range is large enough and the orbit may change a lot after the maneuver. The orbital element set before the maneuver can not be used to predict the orbit after the maneuver, which will cause the maneuver to be unable to be identified. If the low thrust orbit transfer is used, the influence of the low thrust can be compared and analyzed with the influence of the usual perturbation force, the orbit changes very slowly and new orbital elements can be correlated with the correct mother satellite. Analysts can correlate the orbital elements sets before maneuver and those after maneuver, however, the following factors must be taken into considered during the correlation:

1. With the variation of the orbit altitude, the magnitude of each perturbation force also changes ceaselessly, thus, the influence on the orbits of the spacecraft is also different. The spacecraft in LEO is mainly influenced by the terrestrial non-spherical gravitational perturbation and the atmospheric drag, other perturbation factors are all of higher order and small quantity. When the orbit altitude is relatively low (less than 600km, for example), the influence of the solar/lunar perturbation and solar light pressure can be neglected; when the orbit altitude increases (more than 800km), the influence of atmospheric drag can be neglected, or set to be a constant.

2. When the orbit prediction error diverges, the magnitude of the position error is determined by the along-track direction, that of the velocity error is determined by the radial direction; the position error and velocity error oscillates, but does not diverge in the other two directions, the amplitude is generally lower by an order of magnitude. According to the residual average, the different nodes will be predicted as an ephemeris, the along-track oscillation of the position diverges along different directions, some will diverge along the positive direction at the node, and some along the negative direction; the along-track divergence trend of the position is just inverse to that of the radial divergence trend of the velocity.

3. With the increasing of the low thrust, the orbit error caused by the magnitude error and directional error also increases. Each time, when the low thrust increases by an order of magnitude, the orbit error caused by errors in the radial, tangential and normal direction also increase by the same order of magnitude.
References

2. T. S. Kelso. Validation of SGP4 and IS-GPS-200D Against GPS Precision Ephemerides. AAS 07-127