AN ANALYTIC METHOD OF SELECTING POTENTIALLY DANGEROUS OBJECTS FOR COLLISION AVOIDANCE

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ABSTRACT
With the increase of space debris, the technology of spacecraft collision avoidance has become more and more important. In this paper, an analytic method of selecting objects is researched based on mean elements, which is free from the restriction of the spacecraft’s launch time and the instantaneous position. If only the spacecraft’s launch orbit is known, this method can be used to select potentially dangerous objects from thousands of space objects several days before the launch date of the mission.

1. INTRODUCTION
With the continuous development of the space technology and the expansion of the human’s activity in outer space, the number of space debris grows rapidly. And the resulting space collision events also have attracted more and more attention. According to the statistical data, the number of space debris whose dimension is less than 1 cm has increased to several tens of millions, and whose dimension is larger than 1 cm and less than 10 cm has reached more than one hundred thousand, and whose dimension is larger than 10 cm has reached 14000. The space debris, whose dimension is more than 1 cm, can completely damage aircraft. Therefore, the determination of the time for a safe spacecraft launch and a safe flight condition becomes very important in the aerospace mission.

The orbit comparison method is the most directly viewed method used to analyse the spacecraft’s collision. Comparing the orbits of the spacecraft and other space objects can help us find the objects at a distance less than the allowable threshold, and then the collision probability can be calculated. In this method, the relative position of two objects in an epoch should be calculated. Due to the space object moving at a high speed of several kilometres per second, the calculation intervals should be as short as possible for the accuracy. The disadvantage of this method is more obvious for the determination of the time of the safe spacecraft launch. For example, 20 days before the launch date of the spacecraft, if there are 3 launch windows and each window has a 30 minute launch period, it takes about 7 hours for satellite to move from the injection point to the maneuver orbit. Even calculated at an interval of 1 second, the possible launch window will last 5400 seconds, and the operation time of each window will last 27000 seconds, so calculating the relative distance between the spacecraft and hundreds or even thousands objects is impossible. Besides, the relative moving distance between the spacecraft and the object over the interval of 1 second will be more than 10 kilometres. Therefore, the feasible method is to preliminarily filter all the space objects before the orbit comparison, to eliminate those objects that have less threat against spacecraft flight by using some parameters and algorithm, and to select those objects, that may pass through the flight region of the spacecraft, as the potential dangerous objects.

In this paper, an analytic method of objects’ selection based on mean elements is provided, which can select potential dangerous objects from thousands of space objects several days before the launch date of the mission, only the launch orbit of the spacecraft is needed.

2. ANALYTIC METHOD OF OBJECT SELECTION
The basic idea is to first determine two ellipses of the spacecraft and the space objects with known shape and space position, then to solve the minimum space distance of the two ellipses and compare the minimum distance with the allowable threshold, so the potentially dangerous objects can be selected in this way. Because spacecraft and space objects may not arrive at two points with the minimum distance at the same time, the dangerous objects, after selection through the method, may not be real dangerous objects. However, the non-selected objects are absolutely non-dangerous.

It includes four key techniques: preliminary objects selection, allowable threshold determination of collision avoidance, calculation of mean elements, and the solution of the optimization problem.

2.1. Preliminary Objects Selection
Space objects always fly along an ellipse trajectory of different altitudes, so non-dangerous objects can be filtered through the orbit altitude relationship between the spacecraft and space objects. For example, take a
spacecraft with a near-circle orbit of 450km, considering semi-major axis error of 50km, space objects can be divided into three types: objects with height of perigee more than 500km, objects with height of apogee less than 400km, objects with height of perigee less than 500km or height of apogee more than 400km. For the preceding two types of objects, the orbits of the first type is higher and the those of the second type is lower, thus both are non-dangerous objects for the spacecrafts flight, so the potential dangerous objects belong to the third type.

2.2. Allowable Threshold Determination of Collision Avoidance

The allowable threshold of collision avoidance closely relates to the object altitude, orbit determination precision and orbit prediction time. And it varies with the practical situation of the mission.

2.3. Calculation of Mean Orbital Elements

If an instantaneous element at an epoch time \( \sigma_0(a, e, i, \Omega, \omega, M) \) is known, an iterative method is needed to calculate the mean element \([1] \sigma_0\). Assuming the mean element is \( \sigma^{(i)} \) according to the ith iteration, take \( \sigma^{(i)} = \sigma_0 \), use \( \sigma^{(i)} \) to calculate periodic perturbation \( \sigma^{(i)}_p \), further get the ith instantaneous elements \( \sigma^{(i)} = \sigma^{(i)} + \sigma^{(i)}_p \), so the mean elements of the (i+1)th iteration is \( \sigma^{(i+1)} = \sigma^{(i)} + (\sigma_0 - \sigma^{(i)}_p) \). In this paper, only one order short periodic perturbation of main zonal harmonic terms (\( J_2, J_3 \) and \( J_4 \)) and two order short periodic perturbation of tesseral harmonic terms \( J_{22} \) are considered in the transformation between the instantaneous element and the mean element.

2.4. Solution of Optimization Problem

1) Establishment of target function

One of the core techniques to solve the optimization problem is to establish a target function which is the space distance of two points. Assuming the position of the spacecraft at a certain moment is \( r_b(x_b, y_b, z_b) \), the position of any object is \( r(x, y, z) \), so the object function is:

\[
F(r_b, r) = \sqrt{(x-x_b)^2 + (y-y_b)^2 + (z-z_b)^2}
\]  

(1)

2) Relationship between Kepler elements and position

Assume Kepler elements are \((a, e, i, \Omega, \omega, f)\), so the position is

\[
r = r \cos f \hat{P} + r \sin f \hat{Q}
\]  

(2)

where

\[
r = \frac{a(1-e^2)}{1+e \cos f}
\]  

(3)

\[
\hat{P} = \begin{pmatrix}
\cos \Omega \cos \omega - \sin \Omega \sin \omega \cos i \\
\sin \Omega \cos \omega + \cos \Omega \sin \omega \cos i \\
\sin \omega \sin i
\end{pmatrix}
\]  

(4)

\[
\hat{Q} = \begin{pmatrix}
-\cos \Omega \sin \omega - \sin \Omega \cos \omega \cos i \\
-\sin \Omega \sin \omega + \cos \Omega \cos \omega \cos i \\
\cos \omega \sin i
\end{pmatrix}
\]  

(5)

For the target with a certain orbit shape (determined by \( a \) and \( e \)) and orbit position(determined by \( i \), \( \Omega \) and \( \omega \)), \( \hat{P} \) and \( \hat{Q} \) are known, so the position \( r \) is just the function of true anomaly, so \( F \) is the function of \( f \) and \( f_b \).

3) Using the quickly descending method to solve the optimization problem

This optimization problem is non-restricted, formed as follows:

\[
\min_{x \in \mathbb{R}^n} F(f, f_b)
\]

In case of \( F_s(f, f_b) = \text{grad} F(f, f_b) = \left( \frac{\partial F}{\partial f}, \frac{\partial F}{\partial f_b} \right) \),

\[
F_s(f^0, f_b^0) = \text{grad} F(f^0, f_b^0)
\]  

is the grad vector of \( F(f, f_b) \) at the point \((f^0, f_b^0)\). Its norm is

\[
\left\| F_s(f^0, f_b^0) \right\| = \sqrt{\left( \frac{\partial F}{\partial f} \right)^2 + \left( \frac{\partial F}{\partial f_b} \right)^2}
\]

This paper uses the quickly descending method to solve the optimization problem, and iterative steps are:

1) Firstly giving the error range \( \epsilon > 0 \).
II) Selecting an initial point \((f_0^0, f_b^0) \in R^2\) at random, if 
\[ \left\| F_s(x^0, f_b^0) \right\| \leq \varepsilon , \quad \left( f_0^0, f_b^0 \right) \text{ is approximate solution.} \] 
III) If \[ \left\| F_s(f_0^0, f_b^0) \right\| > \varepsilon , \] the method of single variable function to solve extreme value is used to solve \( \lambda_0 \), i.e. 
\[ \min_{\lambda \geq 0} F(x^0 - \lambda F_s(x^0)) = F(x^0 - \lambda_0 F_s(x^0)) \] 
Assuming \( x^1 = x^0 - \lambda_0 F_s(x^0) \). If \[ \left\| F_s(x^1) \right\| \leq \varepsilon \), the iteration can be stopped, \( x^1 \) is taken as approximate solution.

4) Generally, if \( x^k \) is known and \[ \left\| F_s(x^k) \right\| \leq \varepsilon , \] \( x^k \) is the approximate solution. And if \[ \left\| F_s(x^k) \right\| > \varepsilon \), \( \lambda_k \) can be solved by making:
\[ \min_{\lambda \geq 0} F(x^k - \lambda F_s(x^k)) = F(x^k - \lambda_k F_s(x^k)) \] 
Make \( x^{k+1} = x^k - \lambda_k F_s(x^k) \). Repeat the steps above.

If the optimization solution of the problem exists, we shall calculate until getting the approximate optimization solution which satisfies the error range.

3. APPLYICATION IN THE SHENZHOU 7 SPACESHIP MISSION

Generally, the object’s filtration of collision avoidance includes three steps:

1) Filtering related objects using the height exclusion method.
2) Filtering potential dangerous objects using the analytic method.
3) Filtering dangerous objects using the combined method of approach distance and collision probability.

For the shenzhou 7 spaceship on a near-circle orbit of 350 kilometres, considering a proper threshold of collision avoidance, using step 1 and step 2 methods can select related objects and potential dangerous objects; the results are classified as follows:

- Type I Objects on the near circle orbit: \( H_p, H_a \leq 400 km \)
- Type II Objects on the small ellipse orbit \( 400 km < H_a < 3000 km, H_p \leq 400 km \)
- Type III Objects on the big ellipse orbit \( H_a \geq 3000 km, H_p \leq 400 km \)

The related objects after filtration are shown in Tab. 1, and the potential dangerous objects are shown in Tab. 2. According to the two tables, the number of potentially dangerous objects is sharply decreased as compared with related objects. This is mainly because space objects on the big ellipse orbit reduce greatly. Additionally, with the approaching of launch date, the potential dangerous objects will decrease step by step.

<table>
<thead>
<tr>
<th>Table 1 details of related objects (beginning of September)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object classification</td>
</tr>
<tr>
<td>-------------------------------------------------------------</td>
</tr>
<tr>
<td>Type I</td>
</tr>
<tr>
<td>Type II</td>
</tr>
<tr>
<td>Type III</td>
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<tr>
<td>Total</td>
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</tbody>
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<table>
<thead>
<tr>
<th>Table 2 details of potential dangerous objects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Object classification</td>
</tr>
<tr>
<td>------------------------</td>
</tr>
<tr>
<td>Type I</td>
</tr>
<tr>
<td>Type II</td>
</tr>
<tr>
<td>Type III</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

4. CONCLUSION

In this paper, the analytic method of potentially dangerous objects is independent of the instantaneous position of the spacecraft. As a result, either the determination of the time of the safe spacecraft launch or the establishment of the orbit control strategy in the mission, this method is proved to be effective in the objects’ selection of spacecraft collision avoidance.

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