REVEALING LATENT PERIODICITIES IN NOISY SIGNALS

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In [1] theoretical and applied aspects of the new family of selective methods for revealing latent periodicities in signals (called $D$-methods) were expounded. Being introductory into the future publication series on the whole process structure analysis problem, that paper was dedicated to the research of capabilities of $D$-methods when analyzing the “ideal” processes, that is containing finite number of harmonics and, perhaps, some polynomial component, and having no noise.

This paper is accented at the noise problem in application of $D$-methods. The urgency and topicality of the problem is shown and its probability-theoretical description is given. The influence of noise to the quality of revealing polyperiodical structure of processes is investigated. The necessity of perfection of $D$-methods as applied to very noisy signals is substantiated and some approaches to solving this problem are developed. The capabilities of the new methods to be described and published in near future are illustrated with examples of real radar and photometric signatures.

Before addressing the theoretical research of the problem of revealing latent periodicities in noisy signals consider the next simple example of revealing just one harmonic measured with errors. Such a signal is shown at Fig. 1. Its parameters are as follows: amplitude $A = 1$ (in conditional units), period $T = 15$ s, r.m.s. measurement error $\sigma_x = 0.8$, measured time interval $L = 50$ s.

$D_0$-image (the zero order $D$-image) of this signal can be seen at Fig. 2. Because of presence of the $D$-function false minima the harmonic period (its value is placed at the top of Fig.2) is calculated with the error more than 300%. The matter will be worse while dealing with $D$-mapping of higher orders, for instance when trying to separate several close harmonics in presence of noise.

Now let us add another harmonic having amplitude 0.5 and period 22s to the previous signal (Fig. 3). For revealing the least period one needs to construct $D$-image of at least the 1st order. Fig. 4 shows that it is difficult to use the “pure” $D$-methods for selection of periodicities in a very noisy signal especially given at a short time interval.

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Fig. 1. Monoharmonic noisy process

Fig. 2. $D_0$-image of monoharmonic noisy process
Fig. 3 Bi-harmonic noisy signal.

Fig. 4. $D_1$-image of bi-harmonic noisy signal
Now address the analytical estimate of the $D$-mapping characteristics in presence of noise in the discrete task set.

Let the errors of measuring process $X = \{X_k\}$ be Gaussian with the zero expectation and r.m.s. $\sigma_x$. The discrete polyharmonic process looks like

$$X_k = \sum_{i=1}^I A_i \sin\left(\frac{2\pi k}{T_i} + \varphi_i\right) + \xi_k, \quad k = 0 .. N-1,$$

where $N$ – number of measurements,
$A_i$ – amplitude of $i$-th harmonic,
$T_i$ – period of $i$-th harmonic,
$\varphi_i$ – phase of $i$-th harmonic,
$\xi_k$ – error of $k$-th measurement,
$I$ – number of harmonics.

At the outset consider the $D_0$-image of the process. In discrete mode we will have

$$D_0(k) = \frac{\sum_{j=0}^{N-k} |X_{j+k} - X_j + \xi_{j+k} - \xi_j|}{N-k},$$

or

$$\hat{D}_0(k)$$

where $D$ is $D$-image of the noiseless process, $\xi_j = \xi_{j+k} - \xi_j$ - Gaussian random variable.

The distribution of $\xi_j$ has the next parameters:

$$m_{\xi_j} = \frac{2\sigma_x}{\sqrt{\pi}}, \quad \sigma^2_{\xi_j} = \left(1 - \frac{4}{\pi}\right)\sigma^2_x.$$

$$\Delta D_0(k) = D_0(k) - \hat{D}_0(k) \leq \frac{N-k}{N} \sum_{j=0}^{N-k} |\eta_j|,$$

The error of $D_0$-image is limited from above by a quantity
The variance of the error of $D_0$-image is limited from above by a quantity

$$\sigma^2[\Delta \bar{D}_0(k)] \leq \frac{(2 - \frac{4}{\pi})\sigma^2_x}{N - k} \approx \frac{0.7\sigma^2_x}{N - k}.$$ 

Hence, presence of noise in measurements leads to the bias of $D$-image. The amount of the bias is proportional to the r.m.s. error of measurements. The variance of the estimate of $D$-image is proportional to the noise variance and inversely proportional to the measurement interval length.

The bias of the estimate of $D$-image does not influence the accuracy of the period determination because it is not along the absciss axis but normal to that, and for this family of methods only the local minima determination accuracy is important, that is their location at the absciss axis.

The accuracy of determination of periods of the harmonics, and moreover, the possibility of such selection is wholly determined by the variance of $n$-th order, which can be assessed as

$$\sigma^2[\Delta \bar{D}_n(k)] \leq \frac{2\sigma^2_x(1 - \frac{2}{\pi})}{N - k - n} \sum_{i=0}^{n} (\frac{C_i^i}{n})^2, \quad k = 0 .. N-n-1.$$ 

The dependence of the accuracy of period determination on the noise intensity, the $D$-mapping order, the harmonic’s amplitude and period can be derive as follows.

For $D$-method of $n$-th order:

$$D_n(k) = 2A \left( \frac{2\pi}{T} \right)^n \sin\left( \frac{\pi k}{T} \right) \left| \sum_{j=0}^{N-n-k} \cos\left( \frac{2\pi(j+k)}{T} + \varphi \right) \right| \approx \frac{2^{n+2} A\pi^{n-1}}{T^n} \sin\left( \frac{\pi k}{T} \right).$$

At the vicinity of local minima $D$-image has its first derivative by time (by $k$ in discrete case), with the accuracy up to the sign, equal to

$$\frac{dD_n(k)}{dk} = \frac{2^{n+2} \pi^n A}{T^{n+1}}.$$ 

Then the estimate of r.m.s. error of the local minimum abscissa value determination is
Hence, for specified levels of noise and signal it is possible to select the harmonics only given the definite length of the measured time interval. If its span is less than some “critical” value, then the family of “pure” $D$-methods does not fit, to say nothing of all traditional methods. At the same time, a radar or optical signature usually contains an intensive noise component.

To overcome these shortcomings of the known methods, on the basis of selective $D$-methods and autocorrelation functions a new family of methods for revealing latent periodicities were developed for different levels of noise. Their description will be given in posterior publications. Here, some capabilities of these methods are illustrated by the examples of looking for periodicities in real radar and optical signatures. Besides, at Fig. 5 a new “image” of the monoharmonic signal is given. The new method has determined its period $T$ with accuracy of 0.7%.

An optical signature of SO (conditional _1) and its Fourier-transformation are shown at Figs 6 and 7 respectively. One can see that the last one cannot solve our task. However, the new methods reveal the next 7 harmonics:

\begin{align*}
_1 &= 74.58, & _1 &= 4.24; \\
_2 &= 36.06, & _2 &= 9.01; \\
_3 &= 18.26, & _3 &= 2.82; \\
_4 &= 11.99, & _4 &= 4.16; \\
_5 &= 5.833, & _5 &= 1.30; \\
_6 &= 3.256, & _6 &= 1.47; \\
_7 &= 2.783, & _7 &= 1.34.
\end{align*}
Fig. 5. “New” image of monoharmonic noisy signal

Fig. 6. Optical signature — 1
Then the signature \(_1\) was fitted by the generalized polynomial contained all the revealed harmonics and a linear algebraic polynomial. The result of fitting (Fig. 8) eloquently confirms a phenomenological correctness of revealing the poly-periodical structure of the process.
An optical signature (conditional _2) of essentially different kind can be seen at Fig. 9. In this case its Fourier-transformation (Fig. 10) gives much better results, though not enough for sure analysis.
The newest selective methods reveal the next periodicities:

\[
\begin{align*}
_1 &= 6.005, & _1 &= 1.28; \\
_2 &= 2.995, & _2 &= 16.22; \\
_3 &= 1.996, & _3 &= 7.91; \\
_4 &= 1.497, & _4 &= 3.3; \\
_5 &= 1.198, & _5 &= 2.81; \\
_6 &= 0.9979, & _6 &= 2.83; \\
_7 &= 0.8551, & _7 &= 1.81; \\
_8 &= 0.799, & _8 &= 1.45.
\end{align*}
\]

The fitting of signature \_2 by the respective generalized polynomial is presented at Fig.11.

Some radar signatures were analyzed by the new methods. One of them and its Fourier-transformation are presented at Figs 12 and 13 respectively. As one can see, the last cannot be practically used for determination of polyperiodical structure of the process, but only for reference and rather approximate comparison.

![Fig. 11. Correlation of optical signature \_2 and its model](image)

Fig. 12 Radar signature

Fig. 13. Fourier transformation of radar signature
At the same time, the new methods revealed the next periodicities:

\[ t_1 = 817.7, \quad t_1 = 1.48; \]
\[ t_2 = 398.8, \quad t_2 = 1.55; \]
\[ t_3 = 199.0, \quad t_3 = 8.05; \]
\[ t_4 = 132.5, \quad t_4 = 2.57; \]
\[ t_5 = 99.08, \quad t_5 = 3.11; \]
\[ t_6 = 66.23, \quad t_6 = 4.01; \]
\[ t_7 = 56.33, \quad t_7 = 2.11; \]
\[ t_8 = 49.82, \quad t_8 = 1.46; \]
\[ t_9 = 36.09, \quad t_9 = 1.91; \]
\[ t_{10} = 19.06, \quad t_{10} = 2.66. \]

The result of fitting can be seen at Fig. 14.

![Fig. 14. Correlation of radar signature _ 3 and its model](image)

Figs. 15 – 17 present similar results of analysis of another radar signature.
Fig. 15. Radar signature

Fig. 16. Fourier transformation of radar signature
So, the new methods not only show radical improvement of the quality of analysis of polyperiodical structure of radar and optical signatures containing strong noisy components, but in many cases are the only methods capable of correctly solving this complicated problem.

REFERENCES

1. S. Veniaminov, V. Dicky, The New Approach to Revealing Latent Periodicities in Radar and Optical Signatures of Space Objects. (Here.)