A STUDY OF THE SURFACE ENVELOPING THE TRAJECTORY FAMILY OF PARTICLES ISOTROPICALLY EJECTED BY A SATELLITE EXPLOSION WITH ACCOUNT FOR THE ORBIT NODE AND PERICENTER MOTIONS

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INTRODUCTION

An explosion of an artificial satellite leads to the ejection of a mass of fragmentary particles into the cosmic space. Typical velocities of the ejecta are of the order of one km/s or less, so they remain on geocentric orbits $T$ close to the supposedly circukar orbit of the satellite. The emerged swarm fills out a domain $D$ swept by family $\{T\}$. We are interested in studying the structure of the domain $D$, corresponding to the largest possible family $\{T\}$. It leads to an assumption of the ejecta in all directions. Now let’s imagine that we observe an isotropic ejection with all possible velocities smaller than the absolute value $b$.

Due to the inequality of orbital periods, the fragments will densely fill up the domain $D'$. To find its boundary $S'$, it is sufficient to suppose the velocities to be equal to $b$. So, $D'$ represents a dust complex appearing a few days after the explosion.

We have analytically obtained the parametric equations of the boundary $S'$, its properties having been examined in our previous paper [1]. Topologically, $S'$ is a torus with one conic point and one rectilinear constriction (see Fig. 1). In few months $D'$ scatters due to the motion of the nodes and pericenters of particle orbits in the gravitation field of an oblate central planet, which yields the axisymmetric solid $D$ with the boundary $\hat{S}$. Topologically, $D$ is a torus.

Here we have obtained the parametric equations of $\hat{S}$ and examined its properties for the case where the orbit of the exploded satellite lies in the equatorial plane of the central planet.

PROPERTIES OF $S$

Similar to [1], we use the methods of the mathematical theory of catastrophes. The boundary $\hat{S}$ of $D$ is the set of singular points of the mapping $\{T\} \rightarrow \mathbb{R}^3$. Obviously, it is a body of revolution and topologically occurs to be a torus. Hence, it is sufficient to examine its section $S$ by the plane $xz$. 
The mathematical formalism, while obtaining the parametric equations of \( S \), is rather cumbersome, so that we confine ourselves to giving the results.

The curve \( S \) is symmetric with respect to the \( x \)-axis. It consists of the convex oval \( S_0 \) with two additional curvilinear triangles. These triangles are so small that they may be neglected (see Figs. 2 and 3).

The parametric equations of \( S_0 \) are
\[
\begin{align*}
x &= h = h_1 = \\
z &= \theta \in [0, 2\pi]
\end{align*}
\]

with \( c = b \) expressed in units of the circular velocity of the satellite,

The equations are not very complicated. We can easily examine their properties and find the following properties of \( S_0 \).

The curve \( S_0 \) is closed, bounded and symmetric with respect to the \( x \)-axis. The curve \( S_0 \) is analytic (regular) without singular points. The curvature of \( S_0 \) is negative, hence the curve \( S_0 \) is convex.

The family \( \{ S_0 (c) \} \) represents the family of embedded ovals tending to a point as \( c \to 0 \). If \( c \to 0 \), then \( S \) to \( S_0 \) and \( S_0 \) tends to an ellipse with the 1:4 axis-ratio.

CONCLUSIONS

We see that the boundary \( S \) of the section of \( D \) by the plane \( xz \) is an oval. If \( c \) is large, the oval is elongated backwards from the planet. If \( c \) is small, the oval is symmetrical representing the ellipse with the 1:4 axis-ratio.

BIBLIOGRAPHY