Accuracy of orbit determination and prediction for SOs in LEO. Dependence of estimate errors from accuracy and number of measurements

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Outline

1. Comparison of various approaches to SO state vector estimation
2. Technique for estimating the forecasting errors
3. Influence of fit span upon the estimation accuracy
4. Dependence of the optimum fit span from the noise level
5. Dependence of RMS of forecasting errors from the noise level (altitude)
6. Effect of increasing time interval between measurements
7. Effect of increasing the measurement accuracy
8. Effect of the interval of atmospheric disturbances correlation
9. Influence of a number of measurements upon the forecasting accuracy
1. Comparison of various approaches to SO state vector estimation

\[ Z = X \cdot x + B \cdot q + V \]
\[ M \left( V \cdot V^T \right) = \sigma_z^2 \cdot E \]
\[ M \left( q \cdot q^T \right) = \sigma_q^2 \cdot K_q \]

**I Approach.** Without accounting for nuisance (disturbing) parameters.

\[ \hat{x} = \left( X^T \cdot X \right)^{-1} \cdot X^T \cdot Z \]
\[ \hat{y} = \frac{1}{\| \hat{x} \|} = \left( \| X^T \| \| X \cdot B \| \right)^{-1} \cdot X^T \cdot B \cdot Z \]

**II Approach.** Parameterization.

\[ \hat{x} = \left( X^T \cdot P \cdot X \right)^{-1} \cdot X^T \cdot P \cdot Z \]
\[ P = \left( \frac{\sigma_q^2}{\sigma_z^2} \cdot B \cdot K_q \cdot B^T + E \right)^{-1} \]
Dependence of errors from the applied approach

It is seen that there exists a level of (small) disturbances, for which it is more profitably to apply the LST without extension of a state vector. However, even in this case the errors are greater, than in case of using the non-parametric approach.
2. Technique for estimating the forecasting errors

\[ t_{i+1} = t_i + T_i, \]
\[ T_{i+1} = T_i + \Delta T_i, \]
\[ \Delta T_i = \Delta T_m \cdot (1 + q_i). \]

\[ z_i = t_i + v_i \]

\[ K_q(t, \tau)_0 = \left\{ \begin{array}{ll}
\sigma_q^2 \left(1 - \frac{|t - \tau|}{\Delta} \right), & \text{by } |t - \tau| < \Delta, \\
0 & \text{by } |t - \tau| \geq \Delta.
\end{array} \right. \]
Scheme of successive processing of measurements

PARAMETERS: \( dN, \ nz, \ \Delta, \quad S_n = \frac{\sigma_q \cdot \Delta T_m}{\sigma_z} \)
3. Influence of fit span upon the estimation accuracy

Basic parameters:

Time interval between the measurements (in revolutions) $dN=2$;
Interval of atmospheric noise correlation $\Delta = 50$ revolutions;
Mean value of period change per revolution $\Delta T_m = 0.002$ min;
RMS of normalized drag variations $\sigma_q=0.1$.
RMS of time measurement errors $\sigma_z=0.0002$ min $= 0.012$ sec;
The "signal-to-noise merit" $S_n=1.0$

Influence of a fit span upon the accuracy of orbital parameters

<table>
<thead>
<tr>
<th>$n_z$</th>
<th>RMS, sec</th>
<th>RMS, norm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\text{Sig}(0)$</td>
<td>$\text{Sig}(15)$</td>
</tr>
<tr>
<td>4</td>
<td>0.012</td>
<td>0.793</td>
</tr>
<tr>
<td>5</td>
<td>0.011</td>
<td>0.761</td>
</tr>
<tr>
<td>6</td>
<td>0.011</td>
<td>0.775</td>
</tr>
<tr>
<td>7</td>
<td>0.013</td>
<td>0.981</td>
</tr>
<tr>
<td>8</td>
<td>0.016</td>
<td>1.027</td>
</tr>
<tr>
<td>9</td>
<td>0.019</td>
<td>0.930</td>
</tr>
<tr>
<td>10</td>
<td>0.026</td>
<td>1.128</td>
</tr>
<tr>
<td>11</td>
<td>0.039</td>
<td>1.483</td>
</tr>
</tbody>
</table>
4. Dependence of the optimum fit span from the noise level

Estimates of SO drag characteristics at various altitudes
Fit span (number of measurements/day) for SOs at various altitudes and for various accuracy of measurements

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta T_m$, min/rev</td>
<td>2.0E-3</td>
<td>0.6E-3</td>
<td>2.5E-4</td>
<td>1.0E-4</td>
<td>4.0E-5</td>
<td>1.5E-5</td>
</tr>
<tr>
<td>$\Delta T_m$, min/day</td>
<td>0.030</td>
<td>0.009</td>
<td>0.0037</td>
<td>0.0015</td>
<td>0.0006</td>
<td>0.00022</td>
</tr>
<tr>
<td>$\sigma_q \cdot \Delta T_m$, min</td>
<td>2.0E-4</td>
<td>0.6E-4</td>
<td>2.5E-5</td>
<td>1.0E-5</td>
<td>4.0E-6</td>
<td>1.5E-6</td>
</tr>
<tr>
<td>$\sigma_z = 0.0002$, min</td>
<td>5/0.66</td>
<td>6/0.80</td>
<td>8-12/1.33</td>
<td>12-13/1.66</td>
<td>20/2.66</td>
<td>22-26/3.3</td>
</tr>
<tr>
<td>$\sigma_z = 0.001$, min</td>
<td>8-9/1.1</td>
<td>12-13/1.66</td>
<td>16-18/2.3</td>
<td>20-22/2.8</td>
<td>28-30/4.0</td>
<td>42-46/6.0</td>
</tr>
</tbody>
</table>

It is seen from these data, that under considered conditions (dN=2, two versions of RMS errors of measurements) the fit span increases about 5 times, as SO altitudes increase from 300 to 800 km. With 5-fold worsening of measurement accuracy the fit span increases about 1.5-2 times. For the considered versions of initial data the optimum values of a fit span lie in the range from 0.6 to 6 days.
5. Dependence of RMS forecasting errors (sec) from the noise level (altitude)

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>LST by nz=opt</th>
<th>OFM, nz=(15) - 30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sig(0)</td>
<td>Sig(15)</td>
</tr>
<tr>
<td>300</td>
<td>0.011</td>
<td>0.761</td>
</tr>
<tr>
<td>400</td>
<td>0.011</td>
<td>0.320</td>
</tr>
<tr>
<td>500</td>
<td>0.0099</td>
<td>0.172</td>
</tr>
<tr>
<td>600</td>
<td>0.0092</td>
<td>0.090</td>
</tr>
<tr>
<td>700</td>
<td>0.0078</td>
<td>0.046</td>
</tr>
<tr>
<td>800</td>
<td>0.0069</td>
<td>0.027</td>
</tr>
</tbody>
</table>

The decrease of atmospheric noise (with increasing SO’s altitude) had greatest effect on the errors of forecasting for 60 revolutions. They decreased by a factor of 75. In this case the RMS errors at the last measurement instant decreased 1.5 times only. It is also seen from the data presented above, that under considered conditions the application of LST (for the optimum fit span) and OFM results in virtually identical accuracy of estimates. It was shown above, that the deviation from the optimum measurement interval results, with using LST, in worsening the accuracy of estimations. Such an effect is absent in case of using the OFM technique.
6. Effect of increasing time interval between measurements (dN=8)

Obtained data show that, at forecasting, the estimates of RMS of errors for LST are in all cases essentially (up to 1.5 - 2 times) greater, than corresponding errors of the OFM technique. We shall pay attention to the fact, that with using the OFM technique the fit span was rather great (16 days), but this fact did not cause worsening the accuracy of estimates of orbital parameters.

<table>
<thead>
<tr>
<th>Altitude, km</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta T_m), min/rev</td>
<td>2.0E-3</td>
<td>0.6E-3</td>
<td>2.5E-4</td>
<td>1.0E-4</td>
<td>4.0E-5</td>
<td>1.5E-5</td>
</tr>
<tr>
<td>(\Delta T_m), min/day</td>
<td>0.030</td>
<td>0.009</td>
<td>0.0037</td>
<td>0.0015</td>
<td>0.0006</td>
<td>0.00022</td>
</tr>
<tr>
<td>(\sigma_q \cdot \Delta T_m), min</td>
<td>2.0E-4</td>
<td>0.6E-4</td>
<td>2.5E-5</td>
<td>1.0E-5</td>
<td>4.0E-6</td>
<td>1.5E-6</td>
</tr>
<tr>
<td>(n_z)</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
</tr>
<tr>
<td>Fit span, days</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>
7. Effect of increasing the measurement accuracy

These data reveal the following general regularities: 1) the growth of errors of determining and forecasting the orbital parameters with worsening the accuracy of measurements, and 2) the lower level of errors with using the OFM technique as compared to application of LST.

Here, whereas for the OFM technique the presented dependencies are monotonous, for LST estimations the RMS of forecasting errors increase as the values fall below 0.0036 sec.
8. Effect of the interval of atmospheric disturbances correlation

RMS of SO forecasting errors (sec) for various values of the atmospheric disturbances correlation interval

<table>
<thead>
<tr>
<th>Δ revolutions</th>
<th>LST by nz=opt</th>
<th>OFM, nz=30</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sig(0)</td>
<td>Sig(15)</td>
</tr>
<tr>
<td>10</td>
<td>0.0112</td>
<td>1.450</td>
</tr>
<tr>
<td>25</td>
<td>0.0114</td>
<td>1.017</td>
</tr>
<tr>
<td>50</td>
<td>0.0117</td>
<td>0.817</td>
</tr>
<tr>
<td>100</td>
<td>0.0112</td>
<td>0.670</td>
</tr>
<tr>
<td>150</td>
<td>0.0118</td>
<td>0.627</td>
</tr>
</tbody>
</table>

These data reveal the general regularities, which are typical for the overwhelming majority of calculation versions: 1) the decrease of errors of determination and forecasting the orbital parameters with increasing correlation interval, and 2) the lower level of errors with using the OFM technique as compared to application of LST. The mentioned regularities are quite expectable.
9. Influence of a number of measurements upon the forecasting accuracy

The increase of a number of measurements up to 60-100 results in essential lowering of errors. With forecasting intervals longer than 1 day (≈15 revolutions) the RMS of errors of the OFM technique is 1.3 - 1.4 times lower, than corresponding results of LST application (by optimum fit span).
Panel of software “LST & OFM errors”

**Common initial data**

<table>
<thead>
<tr>
<th>nq value</th>
<th>dN value</th>
<th>sigq E-6 minutes</th>
<th>sigz E-6 minutes</th>
<th>Signal/Noise</th>
<th>dN predict</th>
<th>jN predict</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>2</td>
<td>50</td>
<td>200</td>
<td>0.250</td>
<td>5</td>
<td>12</td>
</tr>
</tbody>
</table>

**Initial data for LST**

- nz min: 5
- nz max: 15
- New File (dat): new.dat

**Initial data for OFM**

- nz_OFM: 25
- New File (xt): new_xt.dat

**Old output files of LST**

- mkn_2.dat
- MNK_3.dat
- MNK Move.dat
- new.dat

**Old output files of OFM**

- KA1_errors_xt.dat
- new_xt.dat
- OFM 2_xt.dat
- OFM 3_xt.dat

**Normalization**

- a) RMS of time errors
  
  \[ \text{RMS}_{\Delta t}^{(N)}_{\text{norm}} = \frac{\text{RMS}_{\Delta t}^{(N)}}{\sqrt{0.5 \cdot \sigma_\Delta t \cdot N^2}} \]

- b) RMS of drag (Δt) values
  
  \[ \text{RMS}_{\Delta t}^{\Delta t}_{\text{norm}} = \frac{\text{RMS}_{\Delta t}^{\Delta t}}{\Delta t_{\text{norm}}} \]

**RMS(Npred) and RMS(N0) of normalized errors vs nz**

**RMS of normalized prediction errors**

**Prediction interval, revolutions**
Conclusions

1. A comparative analysis of the accuracy of determination of SO’s state vector with accounting for nuisance (disturbing) parameters was performed with various approaches to the problem solution:

   - By means of LST **without accounting for nuisance (disturbing) parameters**;
   
   - By means of LST with including disturbing parameters into the state vector (**parameterization**);
   
   - By means of maximum-likelihood method (MLM) on the basis of accounting for statistical characteristics of disturbing parameters (**without parameterization**). The a priori correlation matrix of nuisance parameters is used for “weighing” the measurements without extension of a state vector.

   It is shown that there exists a level of (small) disturbances, for which it is more profitably to apply the LST without extension of a state vector. However, even in this case the errors are greater, than in case of using the non-parametric approach. In all cases, the latter approach provides the best accuracy (optimum filtering of measurements).
2. The statistical model is developed for estimating time errors of
determination and forecasting of orbits influenced by atmospheric
disturbances of color noise type. The basic influencing factors are
considered to be:
   - Standard deviation of measurement errors;
   - Time interval between the measurements;
   - Number of processed measurements (fit span);
   - Standard deviation of color noise;
   - Time interval of color noise correlation.

3. The dependence of errors on basic influencing factors was
studied. In so doing, two approaches to state vector estimations were
considered: 1) with applying LST and without accounting for disturbing
parameters, and 2) on the basis of optimum filtering of measurements
with accounting for statistical characteristics of atmospheric
disturbances. As a result, the estimates of time errors under various
conditions were obtained.

4. The results of performed investigation can be useful for updating
the orbit correction techniques applied in Space Surveillance Systems,
which is characterized by extreme diversity of orbits and measurement
performance modes. This especially concerns small-size SOs, for which
obtaining a great number of measurements is problematic.
References


