CATALOGING WITH AN UPGRADED SPACE SURVEILLANCE FENCE*

Felix R. Hoots*, Geoffrey S. Pierce◦, Lester Ford†, Hugh Hadley◊

The Air Force Space Command maintains a catalog of over 12,000 satellites. More than 275,000 observations are processed daily to determine updated element sets for all satellites. More than half of the total observations are contributed by one resource, the Space Surveillance Fence. Currently the Government is considering a replacement of the fence operating at S-band. The higher frequency will detect many more objects and require specialized new algorithms for cataloging. We have developed a simulation of the fence and used it to demonstrate the capabilities of the fence and the algorithms to create a catalog of newly detected satellites using only the upgraded fence.

INTRODUCTION

Air Force Space Command has a mission to maintain a catalog of all Earth orbiting objects. This catalog of manmade objects currently numbers over 10,000 objects that have been identified with known launches. Another 2,000 objects are also maintained, but their country of origin has not been determined. The majority of the cataloged objects are inert, i.e., debris, rocket bodies, or inactive satellites. The catalog maintenance task has historically been accomplished using an analytical, general perturbations (GP) orbit model. Recent advances in computers have allowed the feasibility of using a numerical, special perturbations (SP) model as well to obtain higher accuracy orbit determinations and predictions.

Whether using general perturbations or special perturbations, the orbit models do not perfectly match the real world, and orbit predictions tend to stray from reality as time goes on. Thus, each orbit must be periodically reinitialized to reconcile it with the actual satellite trajectory. The reinitialization, or update, is typically performed one or more times per day depending on availability of new data. The primary source of data is a network of radar and electro-optical tracking sites scattered around the world. More than 275,000 observations are processed daily to determine updated element sets for all satellites. More than half of the total observations are contributed by one resource, the Space Surveillance Fence.

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The fence was first constructed in 1958 and lies along a great circle inclined approximately 33 degrees to the equator and stretching across the southern United States. Although it has been maintained and upgraded over the years, it is nearing the end of sustainability. Currently the Government is considering whether to do a refurbishment to provide service life extension, to completely replace the fence, or to just discontinue its use. In order to assist in this assessment, we have constructed a high fidelity simulation of the fence coupled with the associated orbit catalog maintenance.

In a previous paper\(^1\) we have reported on the details of the simulation model and demonstrated its capabilities. In particular, catalog maintenance on a representative set of 500 satellites was performed for a 30-day period using only the simulated observations from the Space Surveillance Fence. We found that the existing fence alone has a robust capability, successfully maintaining a catalog of all low altitude satellites in our simulation.

In this study we look at a proposed replacement of the fence operating at S-band. Figure 1 depicts the complete fence where transmitters are indicated in red and receivers are indicated in blue.

Figure 1. The S-band space surveillance fence provides coast-to-coast coverage.

At the higher S-band frequency, the fence will see much smaller objects than the remainder of the space surveillance system. It is estimated that the S-band fence will
detect an additional 100,000 orbiting objects. The fact that only the fence will detect these new objects provides a challenging problem for creating an initial catalog entry. One concept for the fence design uses a dual beam in the eastern portion of the fence. This dual beam allows estimation of an initial orbital element set that can later be confirmed as the satellite passes through the fence on a subsequent orbit. Figure 2 provides an end on view of the dual fence in the eastern portion of the United States.

![Dual Beam Fence](image)

Figure 2. The dual beam feature allows initial orbit estimation.

In order to perform initial cataloging using the dual beam fence, it has been necessary to develop two specialized algorithms. The first algorithm performs correlation of detections between the dual beams of the fence. From a pair of correlated detections, an initial orbit estimate is made. Because of the short arc and sparse data from the eastern fence, it is necessary to also develop a specialized algorithm for observation association and orbit validation on the subsequent orbit.

We use our simulation to show that the dual fence design, along with these specialized processing algorithms, facilitates creation of a new satellite catalog using only the upgraded space surveillance fence. New satellites are entered into the catalog and their orbits confirmed at a rate that initially increases exponentially and rapidly builds a catalog of satellites.
SIMULATION OVERVIEW

The fence system simulation contains three main sections: simulation of the observations, new object creation, and catalog maintenance using these simulated observations. These three sections and the associated flow of data are depicted in Figure 3. The following discussion provides a top-level overview of the processing that occurs in each section. Then we will give details of each element of the modeling.

Coverage and Sensitivity Section

A simulation must contain both a ground truth and an attempted reconstruction of that ground truth by the models employed. The difference between the ground truth and
the reconstruction provides the assessment of the performance of the models in the real world. The ground truth begins with a satellite element set and radar cross-section from the NASA debris catalog. This element set is predicted ahead in time for the entire simulation period. This trajectory serves as the ground truth for the satellite position and velocity at any time of interest. The trajectory is then compared to the fence geometric model to determine the time and location of each crossing of the fence throughout the simulation period. The details of the fence-crossing encounter are passed to the radar simulation.

The radar model considers both geometry and radar cross section to determine if there is sufficient return signal strength to obtain a detection. If so, the radar model provides an estimate of the location of the satellite and the time of the fence crossing. This information is converted into a standard fence observation expressed in direction cosines and is passed to the Catalog Maintainability Section for use in updating the orbital element set. Also, the direction cosine observation is compared to the truth to assess whether the radar simulation is creating observations with an appropriate amount of noise. The aggregate statistics from all observations for a given simulation run are used as weights in the orbit determination process.

**Catalog Maintainability Section**

The noisy observations produced by the simulation are not tagged with the true satellite number. The untagged observation first undergoes an association process to determine with which satellite in the catalog the observation most likely belongs. The association process is one of the two new algorithms mentioned in the introduction and is described in detail below. If the observation successfully associates, then it is used to update the satellite orbital element set. Each time the element set is updated, the newly determined orbit is used to predict ahead to provide an update of predicted fence penetration times. These fence penetration times are used subsequently in the association process of further observations.

**New Object Creation Section**

If an observation does not associate, then it most likely belongs to a satellite that is not yet a member of the catalog. If such an observation comes from the dual beam section of the fence, then it may be possible to create an initial estimate of the orbit. The initial orbit estimate is entered into the catalog and is used to predict future fence penetrations. If further observations associate with the new catalog entry, then the orbit can be refined and should be maintainable by the Catalog Maintenance Process. The key challenge here is to correlate the fence penetrations of each of the separate planes of the dual beam fence with one another. The details of this algorithm are discussed below.

**SIMULATION DETAILS**

The following provides details of each step in the simulation processing.
NASA Debris Catalog

NASA has developed a catalog\(^2\) of the expected population of satellites as a function of object size. This model is based on the existing cataloged population of real world satellites with an extrapolation to smaller, uncataloged objects based on historical satellite breakups and theoretical particle size distribution. The catalog contains over 140,000 distinct objects ranging in size from tens of square meters down to one square centimeter. In addition to size distribution, the objects are distributed in three-dimensional space in a manner consistent with the current satellite population, known breakup regions, and future projected catalog growth.

Truth Trajectory

The truth trajectory provides a ground truth for driving the radar model as well as a basis for comparing the estimated orbit with the true satellite location. The simulation has a capability of generating the truth trajectory with a choice of three different models. The simplest is a two-body model. This is useful for assessing the impact of only radar noise and missed detections on the orbit determination and maintenance processing. On the other end of the spectrum, the simulation can generate a truth trajectory using an 8\(^{\text{th}}\) order Gauss-Jackson numerical integration with a 12\(^{\text{th}}\) order Earth geopotential, Jacchia 70 dynamic atmosphere, and lunar and solar gravitation. This model will demonstrate the impact of sparse observations in maintaining orbits that evolve due to perturbations. Finally, the truth trajectory can be generated using the U.S. Navy analytical model\(^3\), PPT3. This model contains the gravitational effects of zonal harmonics through \(J_5\), an empirical atmospheric drag model, as well as lunar and solar gravitational perturbations. We have found that this model contains sufficient perturbations to reveal all pertinent capabilities and limitations of the fence, while requiring considerably less computation time than the numerical integration model.

Fence Crossing

The main fence is modeled with a system of 3 transmitters and 4 receivers. These are located along a great circle described in the WGS84 geophysical coordinate system\(^4\). The origin of the great circle is slightly offset from the origin of the geocentric coordinate system and is located at

\[
X = -14.929 \text{ km} \\
Y = -74.671 \text{ km} \\
Z = 0 \text{ km}
\]

The unit normal to the great circle plane is defined by the unit vector

\[
\hat{i} = 0.10901032 \\
\hat{m} = 0.54526018 \\
\hat{n} = 0.83114865
\]
resulting in a plane whose peak is at a geodetic latitude of 33.96178 degrees. The locations of the sites are given in Table 1.

Table 1. Fence receiver and transmitter locations.

<table>
<thead>
<tr>
<th>Site name</th>
<th>Role</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>San Diego</td>
<td>Receiver</td>
<td>32.57766</td>
<td>-116.97386</td>
<td>8.63682</td>
</tr>
<tr>
<td>Gila River</td>
<td>Transmitter</td>
<td>33.11165</td>
<td>-112.03086</td>
<td>5.93962</td>
</tr>
<tr>
<td>Elephant Butte</td>
<td>Receiver</td>
<td>33.44831</td>
<td>-106.99833</td>
<td>3.16180</td>
</tr>
<tr>
<td>Kickapoo Lake</td>
<td>Transmitter</td>
<td>33.55376</td>
<td>-98.76299</td>
<td>358.58643</td>
</tr>
<tr>
<td>Silver Lake</td>
<td>Receiver</td>
<td>33.14941</td>
<td>-91.02096</td>
<td>354.30240</td>
</tr>
<tr>
<td>Jordan Lake</td>
<td>Transmitter</td>
<td>32.65654</td>
<td>-86.26366</td>
<td>351.70258</td>
</tr>
<tr>
<td>Tattnall</td>
<td>Receiver</td>
<td>32.04238</td>
<td>-81.92609</td>
<td>349.36724</td>
</tr>
</tbody>
</table>

In addition to the main fence that runs coast to coast, we define an auxiliary fence in the eastern United States. This fence is inclined 6 degrees from the vertical in a southerly direction and consists of two receivers and one transmitter. The locations and orientations of the components are provided in Table 2.

Table 2. Auxiliary fence - receiver and transmitter locations.

<table>
<thead>
<tr>
<th>Site name</th>
<th>Role</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Azimuth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Silver Lake</td>
<td>Receiver</td>
<td>33.14941</td>
<td>-91.02096</td>
<td>353.89615</td>
</tr>
<tr>
<td>Jordan Lake</td>
<td>Transmitter</td>
<td>32.64263</td>
<td>-86.26367</td>
<td>351.71903</td>
</tr>
<tr>
<td>Tattnall</td>
<td>Receiver</td>
<td>32.04238</td>
<td>-81.92609</td>
<td>349.77352</td>
</tr>
</tbody>
</table>

The times of fence crossing are obtained by stepping the ephemeris along until the satellite passes from one side of the fence to the other. When this occurs, the exact time of the fence crossing is obtained by iteration. This crossing time and the associated fence direction cosines and bi-static range rate are recorded as the truth observation.

**Radar Model**

The purpose of the radar model is to operate on the true satellite location data and produce realistically errored measurements, representative of target detection reports that would be obtained from the actual fence. Figure 4 illustrates the radar model functions, as they relate to the rest of the fence model.
The postulated S-band fence is a multistatic radar system consisting of three transmitters and four receivers, interleaved from coast to coast along a great circle, roughly at 33 degrees north latitude, and using the same sites as the currently deployed VHF Fence. The western portion of the system, consisting of two transmitters and three receivers, operates in the same way as the current VHF Fence. Any single target detection from a transmitter-receiver pair is sufficient to determine the angle and radial velocity of the target object, but simultaneous detections from two receivers are necessary to locate that object in three dimensions. All target detection reports are autonomous at the receiver sites; hence target localization cannot be performed at the separate sites. All multiple-site data association and tracking functions are performed at a central processing facility in Dahlgren, VA.

The eastern portion of the system, consisting of one transmitter and two receivers, operates in a different way. A shortcoming of the current VHF Fence is that it cannot originate an orbit description on a new object, because it cannot measure proper velocity. The eastern S-band system corrects this deficiency using a dual-beam antenna system, which can measure true velocity and, hence, initiate new catalog entries by predicting the intercept time of the next orbital pass. For existing cataloged objects, the eastern transmitter and receivers simply add to the coverage of the western component. For the combined subsystems, a large high-altitude object can give rise to as many as 12 detection reports (3 transmitters x 4 receivers).

In addition to the ability to perform single-pass orbit prediction, the proposed S-band fence has much better detection sensitivity for smaller (<1 meter) objects, because of operation in the Fraunhofer (or optical) RCS region, rather than the Rayleigh region. For objects with dimensions greater than about 30 cm, the S-band detection sensitivity is roughly equal to that of the VHF Fence; when the object size is decreased to 5 cm, the S-band fence enjoys 30 dB better sensitivity than the VHF Fence. When operationally deployed, an S-band fence would generate and maintain a much larger catalog of smaller objects than now exists. For equal target signal-to-noise ratio (SNR), the radar measurement accuracy (angle, Doppler, epoch time) of the S-band system is approximately the same as that of the VHF Fence.

The radar model is not a simulation; rather, it is an analytical error model. For any event, the host model designates one transmitter-receiver pair, and one orbiting object. The event geometry is specified in radar coordinates; object range, radial velocity, and radial acceleration with respect to each of the two sites, along with the time the object is expected to enter and leave the radar surveillance beam. In addition, the size of the object is given, so that a radar cross-section can be computed. This information is processed in the sequence shown in Figure 5.
The radar figure of merit is determined by its transmit power, antenna gains, and losses and gains in signal processing. The target dynamics determine how long the object will be illuminated by the fence beam. Once the radar-target interaction is completely defined, the pre-detection signal/noise ratio (SNR) can be calculated. Note that bi-static range is computed in order to estimate path loss and SNR, although target range is neither measured nor reported by any single transmitter – receiver pair. Using the predicted SNR, an errored radar observation is constructed, that includes:

- Probability of Detection: A statistical model is used to compute the probability that the target will be detected. This probability of detection (PD) is always reported, along with SNR. Based on a statistical draw at that probability, either the other target parameters will be reported, or a “no detection” will be declared.
- Range Rate: A sample is drawn from a Gaussian population having the correct mean (no bias error) and a variance that varies inversely with the radar SNR.
- East-West Angle: A sample is drawn from a Gaussian population having the correct mean (no bias error) and a variance that varies inversely with the radar SNR.
- Epoch Time: The time at which the target crosses the center of the Fence beam is also reported by a statistical draw having the appropriate variance.

**Observation Correlation**

If an uncataloged satellite crosses both sections of the dual beam fence and produces detections by at least two receivers within each fence plane, then we have sufficient data to correlate the two crossings and create an initial orbit estimate. Figure 6 is representative of one of the dual fences.
where

\[ R_1 = \text{first receiver} \]
\[ \rho_1 = \text{range to first receiver} \]
\[ \dot{\rho}_1 = \text{range-rate to first receiver} \]
\[ R_2 = \text{second receiver} \]
\[ \rho_2 = \text{range to second receiver} \]
\[ \dot{\rho}_2 = \text{range-rate to second receiver} \]
\[ T = \text{transmitter} \]
\[ \tau = \text{range to transmitter} \]
\[ \dot{\tau} = \text{range-rate to transmitter} \]
\[ D = \text{baseline distance between the two receivers} \]
\[ v = \text{velocity in fence plane} \]

Note that these angles are relative to the chord, not the local horizon. For the sake of convention, the westernmost of the pair of receivers will be designated receiver one and the easternmost will be designated receiver two.

**Fence Geometry**

The locations of the transmitters and receivers are specified in geodetic coordinates. Let \( x'_1, y'_1, z'_1 \) and \( x'_2, y'_2, z'_2 \) be the corresponding Earth fixed coordinates of the two receivers. Then the chord distance between the two receivers is

\[
D = \sqrt{(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2}
\]  

(1)
Construct a unit vector from receiver one to receiver two.

\[
\vec{d} = \frac{x_2' - x_1'}{D} \hat{i}' + \frac{y_2' - y_1'}{D} \hat{j}' + \frac{z_2' - z_1'}{D} \hat{k}'
\]  

(2)

Let \( \vec{u}_1 \) and \( \vec{u}_2 \) be unit vectors in ECF coordinates in the direction of the observation from receiver one and two, respectively. Then compute the angles relative to the chord.

\[
\alpha = \cos^{-1}\left(\vec{d} \cdot \vec{u}_1\right)
\]

\[
\gamma = \cos^{-1}\left(\vec{d} \cdot \vec{u}_2\right)
\]

(3)

**Triangulation**

Using the law of sines, one can show that

\[
\rho_1 = D \frac{\sin \gamma}{\sin(\gamma - \alpha)}
\]

\[
\rho_2 = D \frac{\sin \alpha}{\sin(\gamma - \alpha)}
\]

(4)

**Observation Position**

The direction cosines from receiver one along with the radial distance \( \rho_1 \) can be used to compute a position in Earth fixed coordinates. Similarly, the information from the second receiver can be used to compute a second estimate of position in Earth fixed coordinates. These two Earth fixed position estimates are averaged to give the final estimate of Earth fixed position. Let this final position estimate be \( x'_{Sat}, y'_{Sat}, z'_{Sat} \).

**Velocity in the Fence Plane**

Compute the distance from the transmitter to the satellite.

\[
\tau = \sqrt{(x'_{Sat} - x'_T)^2 + (y'_{Sat} - y'_T)^2 + (z'_{Sat} - z'_T)^2}
\]

(5)

where \( x'_T, y'_T, z'_T \) are the Earth fixed coordinates of the transmitter. Compute a unit vector from the transmitter to the satellite.

\[
\vec{l} = \frac{x'_{Sat} - x'_T}{\tau} \hat{i}' + \frac{y'_{Sat} - y'_T}{\tau} \hat{j}' + \frac{z'_{Sat} - z'_T}{\tau} \hat{k}'
\]

(6)
Compute the angle between the chord unit vector and the line from the transmitter to the satellite.

\[ \beta = \cos^{-1} \left( \hat{t} \cdot \hat{d} \right) \]  

(7)

Assume that the two receivers each measure a bi-static range-rate, say \( BRR_{11} \) and \( BRR_{12} \), respectively. Then

\[ \rho_1 + \dot{\nu} = BRR_{11} \]
\[ \rho_2 + \dot{\nu} = BRR_{12} \]  

(8)

This provides two pieces of information about the satellite velocity in the fence plane. Let

\[ \nu = \text{magnitude of velocity in the fence plane} \]
\[ \varepsilon = \text{orientation angle for velocity in the fence plane (relative to } \hat{d} \text{)} \]

From Figure 6, we can resolve the velocity into its components along \( \rho_1, \tau, \rho_2 \). Then

\[ \dot{\rho}_1 = v \cos(\alpha - \varepsilon) \]
\[ \dot{\tau} = v \cos(\beta - \varepsilon) \]
\[ \dot{\rho}_2 = v \cos(\gamma - \varepsilon) \]  

(9)

Using Equations (9) in Equations (8) gives

\[ v \cos(\alpha - \varepsilon) + v \cos(\beta - \varepsilon) = BRR_{11} \]
\[ v \cos(\gamma - \varepsilon) + v \cos(\beta - \varepsilon) = BRR_{12} \]

Solving for \( v \) and \( \varepsilon \) (use two argument arc tangent) gives

\[ \tan \varepsilon = \frac{BRR_{12} \left( \cos \alpha + \cos \beta \right) - BRR_{11} \left( \cos \gamma + \cos \beta \right)}{BRR_{11} \left( \sin \gamma + \sin \beta \right) - BRR_{12} \left( \sin \alpha + \sin \beta \right)} \]  

(10)

and

\[ v = \frac{BRR_{11}}{\cos(\alpha - \varepsilon) + \cos(\beta - \varepsilon)} \]  

(11)

From the values \( v \) and \( \varepsilon \), we can solve for \( \dot{\rho}_1 \) and \( \dot{\rho}_2 \) with Equation (9). Now from the definition of range rate, we can write

\[ \left( \vec{V}_{sat} - \vec{V}_{sensor} \right) \cdot \vec{u}_1 = \dot{\rho}_1 \]
\[ \left( \vec{V}_{sat} - \vec{V}_{sensor} \right) \cdot \vec{u}_2 = \dot{\rho}_2 \]  

(12)

where

\( \vec{V}_{sat} = \text{velocity of the satellite as measured in the inertial frame} \)
\[ \vec{V}_{\text{sensor}} = \text{velocity of the sensor as measured in the inertial frame} \]
\[ \vec{u}_j = \text{unit vector from the sensor to the satellite as measured in the inertial frame} \]

Then combining Equations (9) and (12) gives

\[ \begin{align*}
(\vec{\dot{V}}_{\text{sat}} - \vec{V}_{\text{sensor}}) \cdot \vec{u}_1 &= v \cos(\alpha - \varepsilon) \\
(\vec{\dot{V}}_{\text{sat}} - \vec{V}_{\text{sensor}}) \cdot \vec{u}_2 &= v \cos(\gamma - \varepsilon)
\end{align*} \]

(13)

where \( v \) and \( \varepsilon \) are given by Equations (10) and (11).

Now Equation (13) is two equations containing three unknowns, \( V_x, V_y, V_z \). At this point we assume that the satellite is in a circular orbit with semimajor axis given by

\[ r_1 = \sqrt{x'^2 + y'^2 + z'^2} \]

Then the circular velocity of the satellite is

\[ V = \sqrt{\frac{GM}{r_1}} \]

(14)

where \( GM \) is the gravitational force constant. Then we can express any one component of the velocity in terms of the other two. For example,

\[ V_z = \sqrt{V^2 - V_x^2 - V_y^2} \]

(15)

Combining Equations (13) and (15) gives two equations and two unknowns. Although these equations are non-linear, we can use an iterative technique such as Newton’s method to solve for velocity.

Second Fence Observation Correlation

Let \( \vec{r}_1 \) be the first fence observed satellite location expressed in ECI coordinates. Since we have both a position and a velocity estimate, we can form an orbit. In a dense satellite environment there will be more than one observation in the second fence that could possibly correlate with the observation from the first fence. We use the first fence observation position and computed velocity to predict the time that the orbit intersects the second fence. We consider all second fence observations that occurred within a nominal time of this predicted intersection time.

For each second fence observation, there is a unique orbit that joins the observation from the first fence with the candidate observation in the second fence. We form the residuals of the observations from both fences with the candidate orbit and consider the ratios.
for each component of the observation. Any second fence candidate observation that has any component ratio greater than 10% is rejected. If more than one candidate second fence observation meets the above criteria, then we compute a score

\[
Score = \sum_i \frac{|Observed_i - Predicted_i|}{Observed_i}
\]

(17)

The candidate second fence observation with the lowest score is determined to be the observation to be properly correlated with the first fence observation. In practice, the bistatic range rates are a very effective discriminator for selection of the correct second fence observation.

**Initial Orbit**

Let the ECI coordinates of the two correlated fence penetrations be given by

\[\begin{align*}
\vec{r}_1 &= \text{position vector of first fence penetration} \\
t_1 &= \text{time of first fence penetration} \\
\vec{r}_2 &= \text{position vector of second fence penetration} \\
t_2 &= \text{time of second fence penetration}
\end{align*}\]

Compute inertial velocity by assuming that the position vectors at the two fences can be related by a Taylor series

\[
\vec{r}_2 = \vec{r}_1 + \vec{v}_1 \Delta t + \frac{1}{2} \ddot{a}_1 (\Delta t)^2
\]

(18)

where

\[
\Delta t = t_2 - t_1
\]

\[
\ddot{a} = -\frac{GM}{r_1^3} \vec{r}_1
\]

and the acceleration is the two body acceleration at radius \(r_1\). We can solve for the unknown velocity to get

\[
\vec{v}_1 = \frac{\vec{r}_2 - \vec{r}_1}{\Delta t} - \frac{1}{2} \ddot{a}_1 \Delta t
\]

(19)

So the method produces an initial state vector \((\vec{r}_1, \vec{v}_1)\) with epoch time \(t_1\) for initial entry of the satellite in the catalog.

**Observation Association**
When a fence crossing occurs at some particular time, it produces an observation. This observation must be compared with each of the previously catalogued element sets to determine the satellite with which it associates or that it possibly fails to associate with any of them.

The observation association is carried out in two phases by comparing the observation with each element set in the catalog followed by selection of the element set that most closely matches the observation.

**Representation of observations**

For a given observation, let

- \( \tilde{R}_r \) = location for the radar receiver, in ECF coordinates
- \( \tilde{R}_t \) = location for the radar transmitter, in ECF coordinates
- \( \tilde{O} \) = observation

The components of the observation are

- \( O_1 = t \) = time of the observation
- \( O_2 = EW \) = EW direction cosine of the radar measurement, horizon coordinates
- \( O_3 = \rho \) = range between receiver and target
- \( O_4 = \dot{\rho} \) = bi-static range rate
- \( O_5 = NS \) = NS direction cosine of the radar measurement, horizon coordinates

Note that the range component is not measured for our radar concept. However, for the sake of providing a generalized algorithm description, we include it here. Our analysis does not assume nor use a range measurement. The standard deviations of the measured quantities, assumed independent, are denoted by \( \tilde{O}_{\text{Sig}} \).

**Representation of candidate**

From prior cataloging of element sets, we will have a collection of predicted fence penetrations. Each of these will be of the following form:

- \( t \) = time of the predicted crossing
- \( \tilde{r} \) = satellite position at time of fence crossing, ECF coordinates
- \( \tilde{v} \) = satellite velocity at time of fence crossing, ECF coordinates
- \( C'' \) = 6 x 6 covariance matrix of the position and velocity

**Pairing of observations with candidates**
Based on the prediction, we can calculate $\tilde{O}^{\text{pred}}$, as the five-vector of values we should have measured, if this candidate were in fact correctly associated to the observation. Let

$$
\begin{align*}
\tilde{\rho}_R &= \hat{\nu} - \bar{R}_R \\
\tilde{\rho}_T &= \hat{\nu} - \bar{R}_T \\
\rho_R &= |\tilde{\rho}_R| \\
\rho_T &= |\tilde{\rho}_T| 
\end{align*}
$$

(20)

Then

$$
\begin{align*}
O_1^{\text{pred}} &= t - \frac{\tilde{\rho}_R \cdot \tilde{U}}{\tilde{\nu} \cdot \tilde{U}} \\
O_2^{\text{pred}} &= \frac{\tilde{\rho}_R \cdot \tilde{E}}{\rho_R} \\
O_3^{\text{pred}} &= \rho_R \\
O_4^{\text{pred}} &= \frac{\tilde{\rho}_R \cdot \tilde{v}}{\rho_R} + \frac{\tilde{\rho}_T \cdot \tilde{v}}{\rho_T} \\
O_5^{\text{pred}} &= \frac{\tilde{\rho}_R \cdot \tilde{N}}{\rho_R}
\end{align*}
$$

(21)

where $\tilde{E}, \tilde{N}, \tilde{Z}$ describes the East-North-Up unit vectors in standard horizon coordinates at the receiver.

Let $J$ be the 5 x 6 matrix of partial derivatives of $\tilde{O}^{\text{pred}}$ with respect to the components of the state vector $\tilde{X}$, i.e. $J_{i,j} = \partial O_i^{\text{pred}} / \partial X_j$. Then the matrix of $\tilde{O}^{\text{pred}}$ is

$$
C^{\text{Opred}} = J * C^{tv} * J^T
$$

(22)

The total 5 x 5 covariance matrix, $C^{\text{Total}}$, will then be obtained by adding the predicted covariance $C^{\text{Opred}}$ to the variances of the observations themselves.

$$
C_{i,j}^{\text{Total}} = C_{i,j}^{\text{Opred}} + O_{\text{Sig}_i} * \delta_{i,j}
$$

Residues for this Pairing

The residues of the measurements are now given by
\[
\begin{align*}
\text{Res}_1 &= t - O_1^{\text{pred}} \\
\text{Res}_2 &= EW - O_2^{\text{pred}} \\
\text{Res}_3 &= \rho - O_3^{\text{pred}} \\
\text{Res}_4 &= \dot{\rho} - O_4^{\text{pred}} \\
\text{Res}_5 &= NS - O_5^{\text{pred}}
\end{align*}
\]

(23)

The variances on the main diagonal of \( C^{\text{Total}} \) are the squares of the sigma’s of the predicted residues, i.e. \( \text{Sig}_i = \sqrt{C_{i,i}^{\text{Total}}} \).

**Satellite Association Selection**

For each candidate satellite, we form the ratios

\[
\left( \frac{\text{Res}_i}{\text{Sig}_i} \right)^2
\]

(24)

and require that each be within 6 standard deviations, i.e. the ratio should be less than 36. If each residue passes the 6 sigma test, then we form a score for this candidate given by

\[
\text{Score} = \sum_i \left( \frac{\text{Res}_i}{\text{Sig}_i} \right)^2
\]

(25)

The candidate satellite with the lowest score is declared the satellite to be properly associated with the observation.

**Orbit Determination**

Once the fence simulation obtains new observations, the orbital element set must be updated to incorporate this new observational data. We employ a Gauss weighted least-squares process in which a new state vector and drag parameter are determined which best fit the observational data. The details of the orbit determination processing are documented by Danielson\(^6\).

**Automatic Quality Assessment**

For normal maintenance of resident space objects, an assessment is made of the quality of the results at the completion of each orbit determination. If the results pass all quality tests, then the results of the orbit determination are used to update the orbital element set. If the results do not pass all quality tests, then the orbital element set is not updated. Even if the element set is not updated for this observation, it is possible that a subsequent pass through the fence will produce an associated observation that yields an orbit determination that passes all the quality tests. Eight tests make up the complete quality assessment and are given in the paper by Pierce\(^1\).
When cataloging new objects, an initial estimate of the orbit probably will not satisfy all eight quality measures normally applied for an object that has been tracked for a number of days. Rather, the quality metrics used for a stable object will gradually become satisfied as the orbital estimation becomes more stable. For this reason, we have devised a scheme whereby different quality metrics are applied at different stages of the early orbit maintenance process. We transition from the modified quality assessment to the full quality assessment based on the following key parameters: span of available observations, preferred orbit determination interval (ODI), and number of residuals as shown in Table 3.

Table 3. Quality metric application for new objects.

<table>
<thead>
<tr>
<th>Object Type</th>
<th>Available Ob Span</th>
<th>Minimum # Residuals</th>
<th>Quality Assessment</th>
</tr>
</thead>
<tbody>
<tr>
<td>New</td>
<td>seconds</td>
<td>8</td>
<td>None</td>
</tr>
<tr>
<td>Transition</td>
<td>&lt; 1 ODI</td>
<td>8</td>
<td>Modified</td>
</tr>
<tr>
<td>Maintenance</td>
<td>&gt; 1 ODI</td>
<td>18</td>
<td>Full</td>
</tr>
</tbody>
</table>

For the transition stage, the modified quality assessment checks are:
- There must be at least 18 observation residuals accepted by the orbit determination.
- At least 80% of the available residuals must be accepted by the orbit determination.
- The final vector magnitude RMS must be less than or equal to 15 km for near Earth satellites and less than or equal to 25 km for deep space satellites.
- The total plane change resulting from the orbit update must be less than 1.0 degree.

**CATALOGING SIMULATION**

As a means of assessing the capability of the dual fence to create a catalog, we have analyzed its performance using a catalog of element sets representative of the real world in both spatial distribution and range of satellite size. The simulation includes a realistic radar model that assesses probability of detection and provides measurements or observations containing uncertainties reflecting the radar performance. These measurement uncertainties compare closely with the expected real world performance of the proposed Upgraded Space Surveillance Fence.

Although the full NASA debris catalog contains 140,000 objects, it is sufficient to test the fence simulation using a representative subset. We first selected all objects from the NASA catalog having an inclination greater than 32 degrees, since any object with inclination less than 32 degrees will not penetrate any portion of the fence. We randomly selected 525 objects from this reduced NASA catalog to use as our representative sample catalog for testing the fence cataloging capability.
SIMULATION RESULTS

We begin our simulation with 525 satellites orbiting the Earth, none of which are in the satellite catalog. The fence radar simulation produces observations as the satellites pass through the fence. Only observations that pass through both planes of the dual fence in the east can initiate a catalog entry. Whenever a satellite passes through the dual fence, if correlation is successful, an initial catalog entry is made. Provided that subsequent observation associations are successful, the object becomes a maintainable member of the satellite catalog.

The rate at which a catalog could be built from scratch is of great interest, especially since the S-band fence is expected to detect 100,000 objects not in the current catalog. In order to establish a baseline for the best possible performance of adding objects to the catalog, we first ran the simulation assuming perfect dual fence correlation and perfect observation association. Thus, the baseline represents how fast the 525 satellites could be cataloged just based on geometric opportunities to be detected by the fence. We then made the same simulation run, but employed the correlation and association algorithms discussed earlier. This represents the expected performance of the fence and cataloging algorithms in a real world environment. Figure 7 shows the cumulative number of satellites in the catalog as a function of time.

![Figure 7. Catalog growth rate.](image)

Because the dual fence correlation process is not perfect, there will be occasional false orbits created. That is, observations from each plane of the dual fence will be correlated together to form an orbit, but the observations are actually on two different satellites. Thus, the orbit formed will not have any subsequent successful observation associations and, in fact, represent a fictitious orbit. It is also possible to create a legitimate orbit but of such poor quality that there are no subsequent successful observation associations. We expect that the rate of false orbit creation will diminish rapidly as the satellite catalog grows. Figure 8 shows the rate at which false orbits are created over the time span of the simulation.
Finally, there will be some number of observations that do not associate with any object in the catalog. Such observations are called unassociated observations. The primary cause of such observations is satellites being observed by the fence, but not yet having passed through the dual fence to achieve initial catalog entry. We expect that there will be a fair number of these early on in the simulation but the number will rapidly diminish. Figure 9 shows the rate at which unassociated observations occur over the time span of the simulation.

CONCLUSIONS
Any Space Surveillance Fence upgrade concept must include a capability to estimate initial orbits. Since the fence is likely to be the only sensor to view the 100,000 new objects, it must be able to create a catalog autonomously. We have demonstrated how a modest initial orbit estimation capability can lead to a rapid growth of the catalog. Due to the rapid growth potential and the large number of objects, it is essential that the need for human interaction be minimized. The keys to reduction of the man in the loop are:

- The processing must employ “smart” algorithms
- The processing must effectively utilize information such as the element set covariance
- The processing must include tunable rules for automated cataloging, e.g. the quality assessment evolution

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REFERENCES

2NASA 2015 debris catalog derived from 1 Jan 1999 population of 5,116 particles ≥ 2 cm extracted from FY98 Haystack data; includes collisions and explosions.