A Survey of Radars Capable of Providing Small Debris Measurements for Orbit Prediction

Dr. David W. Walsh

Abstract

This paper reviews the basic radar requirements for tracking small debris (1 to 10 cm). Based on results of the NASA ORDERACS experiment the frequency and sensitivity of five current world-wide radars collecting debris data are reviewed. An analysis of the tracking errors of these radars is provided. Based on their range, velocity, and angle track errors and search capability, the Russian Don-2D and the US FPS-85 are identified as the most likely radars to contribute un-cued search and track data to a Space Debris Surveillance Network. The ability of all the radars to provide orbital period data on small debris is assessed. The current criteria to catalog space objects are reviewed in light of NASA Measurements and Modeling of the small debris spatial density. A summary of current radar capability to support a Debris Surveillance Network is provided and recommendations made for future track data experimentation and data exchanges.

Introduction

It is estimated that there are more than 20,000 debris objects with diameters larger than 10 cm (and 600,000 with diameter larger than 1 cm) orbiting the earth (Ref: Air & Space, April 6, 2011) as a result of four decades of space activity. This estimate includes the functioning satellites, but by far the most objects are what are called space debris (SD), man-made orbital objects which no longer serve any useful purpose. Many of the small-sized (less than 10 cm) particles are due to explosions of spacecraft and rocket upper stages and spacecraft collisions, but there are also exhaust particles from solid rocket motors, leaked cooling agents, and particles put into space intentionally for research purposes. The large (> 10 cm) objects have known orbits and are routinely monitored by the U.S. Space Surveillance Network, but information about the smaller particles is fragmentary and mainly statistical.

The current spacecraft shielding, such as that used for the manned modules of the Space Station, are only capable of protecting against debris with diameters below about 1 cm. As their only means of protection, Spacecraft maneuvering is required to avoid collision with debris larger than 1 cm. This, however, requires that the orbit of the debris
object be precisely known. Currently the US Space Surveillance Radars used to track low earth orbit, LEO, objects are only tracking debris like objects down to 5 cm. Of the estimated 600,000 objects above 1 cm, only some 22,000 can be tracked as of today. This leads to wide uncertainties in the estimated quantities of debris, and their predicted orbits. If a collision with larger debris does occur, many of the resulting fragments from the damaged spacecraft will become an additional collision risk.

Given the limited radar detection coverage of objects in the 1 – 10 cm size range and the limited search capability of radars to find and track such objects, limiting the radar search space is critical to their detection. As a result, NASA designed the ODERACS experiments. The ODERACS experiments were efforts to calibrate the radar cross section (RCS) measurements of worldwide radars by launching a series of calibration targets from the space shuttle. Such calibration is critical to the characterization of the debris environment. The radar experiments were also conducted to develop strategies for small debris detection and track. The first experiment launched a series of spheres (5, 10 and 15 cm) from STS-60 at an altitude of 350 km. The X-Band Haystack radar was successful at finding and tracking the 10 and 15 cm spheres. The basic search strategy was to find the spheres based on established orbital elements of the space shuttle. The FGAN Tracking and Imaging Radar (TIRA) was also able to measure and analyze the ODERACS spheres. The Russian Don-2N anti-ballistic missile (ABM) phased array radar also demonstrated its ability to detect and track the ODERACS spheres. The Don-2N was the only world radar able to detect and construct a trajectory of the 5 cm sphere. The FPS-85 phase array radar routinely tracks 10 cm objects to support the US Space Object Catalog. As a result, these radars have demonstrated an ability to support a small debris catalog.

Historically, NASA and the European Space Agency (ESA) have sponsored measurement and modeling efforts to characterize the LEO debris environment. The NASA Orbital Debris Program Office at Johnson Space Center has developed orbital debris engineering models to estimate the orbital debris environment (debris spatial density, flux, etc.). Models such as NASA ORDEM2000 and the ESA MASTER provide a complete description of the environment in terms of debris flux onto spacecraft surfaces or debris detection rate observed by a ground-based sensor. Other models, such as the NASA SBRAM (satellite breakup risk-assessment model) and the NASA EVOLVE (long-term debris evolution model), are more applicable to evaluating the short-term collision risk, due to fragments from recent breakup events, and the long-term impact of various mitigation measures on the debris environment, including secular effects such as the solar activity cycle, which affects atmospheric density and, hence, the decay rate of objects in low Earth orbit (LEO), the growth of the space vehicle population, and a projected fragmentation rate. It should be noted that the NASA and ESA models are not intended to catalog debris (i.e., create debris elements sets for avoidance), but rather to determine the LEO debris population for risk assessment and spacecraft design. The major data sources for the population are:

- The US Space Surveillance Network (UHF and VHF radars) catalog which builds the 1-m and 10-cm populations
- The MIT/LL Haystack (X-band), HAX (Ku-band) and the FGAN/TIRA radar data which build the 1-2 cm population, but do not catalog the population
The LDEF (Long-Duration Exposure Facility) measurements which build the 10-microm and 100-microm populations

The United States Space Command Space Surveillance Network is composed of ground- and space-based sensor systems to track resident space objects. The FPS-85 located at Eglin Air Force Base supplies the majority of the element set and RCS data for LEO objects. The VHF NAVSPUR Fence contributes to the element set data base. These data are compiled daily into Keplerian element sets. In addition to the two-line element data, the radar cross section (RCS)-size data set and the area-to-mass ratio database are also maintained. The NASA Size Estimation Model (SEM) is used to derive size from RCS measurements.

Since there is only very limited element set data available for the 1 to 10 cm population, the ability to avoid debris in this population is extremely limited. In an effort to solve this problem the US Air Force awarded Lockheed Martin and Raytheon $107 million Space Fence Contracts in 2011 to develop an S-band Space Fence to replace NAVSPUR. An award is to be made in 2013 with construction of the first radar on Kwajalein Island to begin in 2013 with Operational Capability in 2017. In addition the US has upgraded the software in the FPS-85 system to test a “Debris Fence”. ESA also plans to add to the arsenal of space debris fighting technology with contracts to Spain and France to design and test two radar concepts. The Spanish test radar located outside of Madrid is a monostatic design with the transmitter and receiver within a few hundred meters of each other. The radar underwent its first acceptance tests in 2012. The French radar will be a bi-static design. The contract signed in 2012 will provided a transmitter and receiver separated some several hundred kilometers around Paris.

Since multi-sensor tracking data are a key element in verifying and updating new object catalog entries, a key issue is the capability of the current LEO radar systems to provide precision tracking data to help catalog the high risk, 1 to 10 cm, population in support of the potential new S-band Fence and related space debris detection and tracking efforts. The ability to improve the track quality of the current radar systems (particularly the range, velocity, and angle measurements) will determine their capability to provide precise orbital period and inclination data on small debris, sufficient for cataloging.

While the U.S. moderates access to information from its SSN, it has expanded its Space Situation Awareness (SSA) Sharing Program. In response to the 2009 Cosmos-Iridium satellite collision, the U.S. military has added personnel and resources to enable it to screen maneuverable, active satellites for potential collisions. As part of this development, the US hopes to expand the number of outside partners and share data about potential collisions. From the end of 2011 to the beginning of the 2012 years the US, Russia and the international community were engaged in international deorbit and reentry campaigns. These campaigns demonstrated the ability of international cooperation to solve the serious problems related to the space area pollution. The response of the international community to participate in the real-time tracking of the failed Phobos-Grunt spacecraft and its deorbit, with the prediction of impact time and the area of its unburnt fragments, is an example of the cooperation and data exchange required to facilitate a small debris catalog. The recent US/Russian Space Surveillance Workshop (August 2012) identified not only the need to share space object catalogs, but the desire to exchange measurement data. Studies have shown the advantage of multiple site data integration in improving the accuracy of orbit prediction and the
possibility of cataloging small debris. The ability to identify likely candidates to provide small debris tracking data is a major element in the creation of a small debris catalog.

**Current Capability**

In assessing the capability of current debris detection radars to generate orbits on space debris a number of issues will be addressed. These include, determining the current radar sensitivity (i.e., the detectable RCS as a function of range) and track capability (i.e., track time, measurement errors). The improvement in track capability/accuracy, that can be achieved by changing the operating modes of the current debris detection radars, will be reviewed, including the reduction of range measurement errors to enhance orbit prediction and ultimately cataloging. Track accuracy is a function of the sensitivity (i.e., signal-to-noise ratio, SNR) of the radar on a given target. The sensitivity is a function of target size as measured by the Radar Cross Section (RCS) of the target. NASA has developed the Size Estimation Model (SEM) to estimate the size of the target based on the RCS. The SEM is derived from multi-frequency measurements of thirty-nine “representative” debris objects selected from two hypervelocity impacts of simulated satellites, and presents the size (estimated diameter) of the objects measured as a function of the measured RCS. Figure 1 shows the relationship for various frequencies, including those for the radars being reviewed. The approximation to the SEM results used here assumes a direct transition from the optical to the Rayleigh region.
The break in the curves differentiates between what nominally is called the Rayleigh region and Optical region. In the Optical region the RCS is independent of frequency. At the break the Rayleigh scattering varies as the sixth power of the object size and the fourth power of wavelength. As shown, the FPS-85, operating at UHF, requires extreme sensitivity to detect and track 1 to 5 cm objects. At the S-band frequency and higher the scattering of 1 to 10 cm objects is still in the Optical region, making the RCS sensitivity requirement significantly less than for lower frequencies. In this region RCS varies as the square of target (debris) size (estimated diameter). A 20 dB improvement in RCS sensitivity at a given range will result in a 10:1 reduction in the debris size that can be detected. Unfortunately, for the FPS-85, which is operating in the Rayleigh region for small debris, the RCS varies as the 6th power of size. A 60 dB improvement in RCS sensitivity is required to achieve a 10:1 reduction in detectable debris size. Thus, for the FPS-85 to detect a 1 cm target requires the radar to achieve a $-70$ dBsm sensitivity, while the X-band and Ku-band, Haystack and HAX, respectively, need only achieve a sensitivity to detect -40 dBsm targets. While the FPS-85 is limited due to its operating frequency, the question now is can improved operating modes (search/track at higher elevations, multiple pulse integration) improve sensitivity and track accuracy to allow the radar to track 1 to 5 cm debris? And can the Haystack and HAX radars, which currently operate with unmodulated pulses in a fixed beam staring mode, change their modes to allow target tracking? And can the Don-2N provide pulse integration to improve search capability? To determine the ability of each of the current debris measuring radars to meet these requirements the operating parameters of each will be reviewed.

**Haystack/HAX**

NASA has been using the MIT/LL Long Range Imaging Radar, known as Haystack, and the Haystack Auxiliary (HAX) Radar to characterize space debris in size, inclination and altitude since 1990. The HAX became operational in 1994 and is used primarily to observe the low earth orbit (LEO) debris environment. Although its sensitivity is lower than Haystack it has a wider field-of-view (1.7 times that of Haystack). The HAX observation mode is currently 75° east. The average debris diameter detected has been reported as from 2 cm to several meters (based on the NASA Size Estimation Model, SEM). Haystack is reported to detect debris from less than 1 cm to several meters. The Haystack/HAX debris detections are of limited quality to determine the particle’s eccentricity accurately. These measurements represent statistical samplings of the population, and are, thus, subject to sampling error.

The Haystack antenna is a 36.6m parabolic reflector, the half power beamwidth is 0.056°. The Haystack pointing accuracy is approximately 1.5 millidegrees. The slew rate of the antenna is 2°/second. The slew rate acceleration is 1.8°/second². The narrow beamwidth and the slow slew rate acceleration are too slow to allow Haystack to provide stare and chase tracks on low SNR debris. A cued search would be required.

The HAX antenna is a 12.2m parabolic reflector, the half power beamwidth is 0.10°. The HAX pointing accuracy is approximately 2 millidegrees. The slew rate of the antenna is 10°/second. The larger beamwidth (smaller reflector) and higher slew rate of HAX radar might be able to support stare and chase tracks on low SNR debris.
Figure 2 is a summary of the Haystack measurements during the 2003 measurement campaign. The large population of debris between 850 and 1000km altitude has been identified as small spherical droplets of eutectic sodium-potassium (NaK) coolant. The NaK coolant leaked from fast neutron reactors that separated from the Russian Radar Ocean Reconnaissance Satellites (RORSATs) at the end of their lifetime. Estimates based on the Haystack measurements indicate that the majority of these objects have an estimated size less than 2 cm. Also the presence of a near-circular debris ring in polar orbit in the 1200 to 1400km altitude region has been observed. Most of the debris objects in this ring are less than 4 cm. The altitude, inclination and observation times of the debris correspond to the orbit plane of the nuclear powered SNAPSHOT satellite which is well known for shedding pieces of debris (more than 50 pieces have been cataloged). Thus, not only must the radars meet the RCS requirements of Fig.1, they must also do so primarily over the altitude region from 500 km to 1200 km as depicted in Fig 2.
FPS-85

The FPS-85 Phased Array Space Surveillance Radar, operational in 1969, is the only US phased array radar dedicated to space surveillance. The radar collects 16 million satellite observations per year. It can detect, track and identify up to 200 space objects simultaneously. It is the only phased array radar capable of tracking deep space objects (can track a basketball size object at 22,000 nm). The bore-sight is at 45°, the nominal low elevation surveillance fence is at 20° elevation. The FPS-85 has upgraded software (1999) to erect a high elevation “debris” fence. Developmental testing of a fence at 35° enabled detection of objects greater than -35dBsm.

TIRA

Europe still has only very limited capabilities for the detection and tracking of ‘uncooperative’ space objects. One high-performance radar facility in Europe able to probe the Earth’s space-debris environment and track and even image space objects is the FGAN Tracking and Imaging Radar (TIRA) system. The FGAN radar is sensitive enough to detect 2 cm sized objects at 1000 km. Because of its unique capabilities, it is used for LEO orbit determination and imaging of re-entering objects. FGAN cooperates with institutes in Germany and abroad, as well as with national and international organizations such as DLR, ESA, NASA and NASDA. With beam-park experiments with TIRA alone, or jointly with the Max-Planck-Institute of Radio Astronomy’s 100 m telescope at Effelsberg in Germany (bi-static mode), snapshots typically of 24 hours duration can be taken of the current space-debris population to provide statistical information and rough orbit parameters for objects as small as 1 cm at altitudes up to 1000 km. In several such experiments, like the NASA Haystack experiments, uncatalogued centimeter-sized debris could be detected, and in some cases the possible sources identified, such as the droplets generated by RORSAT reactor cores and debris from a Pegasus upper-stage explosion (see Figure 2).

Don-2N

The only addition to the Russian Early Warning (EW) Radar System that took place in the late 1980s was the Don-2N (Pill Box) radar of the A-135 Moscow missile defense system, located in Pushkino. Operating at S-band the radar is the highest frequency radar in the EW system. This large (16 meter diameter) phased-array radar reached full operational capability around 1989 and was integrated into the early warning network. Built as a missile defense battle-management radar, Don-2N can cover all elevation or azimuth angles. The radar in its early warning role replaced the old Dunay-3 and Dunay-3U radars that were part of the first Moscow ABM system. The unique capabilities of “Don-2N were clearly demonstrated in 1994 by the results of an international experiment on the detection of small space objects held during the “ODERACS” experiment. In the experiment 3 metal spheres (2, 4, and 6 inches in diameter) were launched from the Space Shuttle (STS 60). The radar was able to detect and track the smallest sphere with some pulse integration. In 2007 a launch of an ABM interceptor was made to test new computational software upgrades to the system.

Radar Parameters
A summary of the radar parameters is given in Table 1. The Haystack/HAX parameters are those reported in the NASA report which summarizes the most recent debris collection campaigns\(^3\). When operating in this mode the radars used an unmodulated CW Pulse of 1.64 msec, with 16 pulse integration. Since the range measurement error is a function of pulse width, range accuracy in this mode is in the tens of kilometers.

<table>
<thead>
<tr>
<th>Radar Parameter</th>
<th>FPS-85 (Trans/REC)</th>
<th>Haystack</th>
<th>HAX</th>
<th>TIRA (L/Ku)</th>
<th>Don-2N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Power (kW)</td>
<td>32000</td>
<td>250</td>
<td>50</td>
<td>2000/13</td>
<td>25000</td>
</tr>
<tr>
<td>Frequency (GHz)</td>
<td>0.442</td>
<td>10</td>
<td>16.7</td>
<td>103/16.7</td>
<td>4</td>
</tr>
<tr>
<td>Beamwidth (deg)</td>
<td>1.3/0.7</td>
<td>0.058</td>
<td>0.10</td>
<td>0.5/0.039</td>
<td>0.27</td>
</tr>
<tr>
<td>Antenna Gain (dB)</td>
<td>43/48</td>
<td>64</td>
<td>67</td>
<td>51/73</td>
<td>57</td>
</tr>
<tr>
<td>Available LFM BW (GHz)</td>
<td>0.001</td>
<td>1</td>
<td>2</td>
<td>0.06/0.8</td>
<td>0.0033</td>
</tr>
<tr>
<td>Pulse Width (msec)</td>
<td>0.25</td>
<td>1.64</td>
<td>1.64</td>
<td>1/0.26</td>
<td>0.0625</td>
</tr>
<tr>
<td>Single Pulse SNR on 0 dBsm @ 1000km (dB)</td>
<td>64</td>
<td>59.2</td>
<td>40.6</td>
<td>51.2/27</td>
<td>45</td>
</tr>
</tbody>
</table>

The radar transmitters do have a high range resolution linear frequency modulation (LFM) mode, 1 MHz bandwidth for Haystack and 2 MHz bandwidth, which would significantly increase the range accuracy.\(^{13}\) In the CW pulse mode it will be assumed that range-rate (Doppler) measurements were made. These measurements can yield velocity measurements, which can be used to estimate orbit inclination more accurately than range-time processing with the very long pulses.

Parameters for the FPS-85 are partially available in Fact Sheets\(^{14}\), articles\(^{15}\) and published reports on the “Debris Fence”\(^{16}\). Based on the number of elements in the transmitter (5928) array and receiver array (19500) the antenna gains and beamwidths shown in Table 1 were computed using the standard equations from Skolnik\(^{17}\). Each element in the transmitter array is driven by a dedicated 0.25msec pulse width (4 kHz
equivalent transmit bandwidth) radar transmitter unit and linear frequency modulation, LFM, (pulse chirping) of up to 1 MHz can be applied to enhance signal processing\(^19\). The single pulse SNR at 1000km was computed using the standard range equation\(^9\). A 4 kHz bandwidth, a loss of 6 dB, and receiver noise temperature of 300° K were assumed in the calculation.

Parameters for the FGAN TIRA radar were taken from the range performance numbers provided in a description of the radar system\(^20\). The main subsystems of the FGAN Tracking and Imaging Radar (TIRA) are: a 34-m parabolic antenna, a narrow-band mono-pulse L-band tracking radar, and a high resolution Ku-band imaging radar. The sophisticated, fully computer-controlled, 34-m parabolic Cassegrain-feed antenna is mounted on an elevation-over-azimuth pedestal. The L-band radar is used primarily for the detection and tracking of space objects. Using a double-Klystron power stage, it generates high-frequency pulses of typically 1 to 2 MW peak power and 1 ms pulse length. The signal processing concept supports target tracking in angular direction as well as in range and range rate. In this operating mode, up to 30 statistically independent observation vectors per second are measured with the tracking filter. The main components of an observation vector are: time, azimuth and elevation angles, range, range rate, echo amplitude and phase, and the transmitted peak power. The Ku-band radar’s main application is in the imaging of space objects, being operated simultaneously with the tracking radar on the same target. Typically, linear frequency modulated radar pulses of 13 kW peak power, 256 microsec pulse length and 800 MHz bandwidth are generated by a travelling wave tube.

Parameters for the Don-2N were primarily taken from a translated description of the radar in Wikipedia (the on-line encyclopedia)\(^21\). Since little is known of the waveforms used in this radar the single pulse SNR at 1000km was scaled from the result presented in the Wikipedia using a detection threshold of 10 dB. The reference cited a target detection range of 1500-2000km on a 0.0019 m\(^2\) target.

**Predicted Radar Performance**

The radar performance for each of the current debris collection radars, in terms of the detectable debris as a function of altitude, was computed using the SNR values at 1000 km, as shown in Table 1. At each altitude the range to that altitude was computed and the SNR scaled to that range using the range to the 4\(^{th}\) power scaling from the standard range equation. For the TIRA, Haystack and HAX radars the range was computed for an elevation angle of 75°. For the FPS-85 and Don-2N phased array radars an elevation of 45° was assumed. This is consistent with raising the current search fence from 25° on boresight to near a boresight “Debris Fence”. A SNR detection threshold of 10 dB was assumed in the calculation. This will result in a high probability of detection, but with a fairly high probability of false alarm. Designing a system to track all small targets will allow for multi-pulse processing which should mitigate the potential false alarm problem.

The results of scaling the SNR values to determine the detectable RCS at various altitudes are shown in Figure 3. The estimated diameter values shown in the figure were determined from the RCS using the SEM model depicted in Figure 1. The sensitivity of the Haystack, TIRA and FPS-85 radars result in detectable RCS values in the SEM Rayleigh region. The sensitivity of the HAX and Don-2N radars, result in RCS values primarily in the optical SEM region. Thus, a slight increase in the sensitivity of either of
these radars (i.e., using multi-pulse integration) can significantly increase the detectable debris size. In contrast, using multi-pulse integration in the FPS-85 will yield only marginal increases in the detectable debris size.

As shown, at maximum sensitivity the FPS-85 can detect a 2.5 cm estimated diameter debris object at 500 km. This is equivalent to detecting a -60 dBsm target at about 516 km with an SNR of 10 dB. At 1000 km a 3.9 cm piece of debris can be detected, which is equivalent to detecting a -49 dBsm target. A 12 dB loss in sensitivity is, thus, equal to a loss of 1.5 cm in detectable debris size. In comparison a 12 dB loss in HAX sensitivity will result in a change of detectable debris size of from 0.8 cm at 500 km to 3.4 cm. Clearly improving HAX or the Don-2N sensitivity by integration or other means has a significant impact on its capability in the debris size region of interest.

Radar Measurement Errors

The precision with which a set of radar measurements, taken while tracking space debris, can produce an orbit for the debris that will depend on the accuracy with which the measurements are made. The radars being evaluated here measure range, angle and velocity. The errors associated with these measurements must be determined before the radars ability to establish an orbit can be assessed.

The radar range measurement error, $\sigma_r$, is generally defined as the root-sum-square of three error components

$$\sigma_R = (\sigma_{RN}^2 + \sigma_{RF}^2 + \sigma_{RB}^2)^{1/2}$$

where the noise range error, $\sigma_{RN} = \Delta R/(2\text{SNR})^{1/2}$, $\Delta R$ is the radar range resolution, which is equal to $c$ (the velocity of light) divided by twice the radar bandwidth; $\sigma_{RF}$ is the
fixed random error due to random noise in the receiver and is equivalent to the noise range error at a SNR of 20 dB; and $\sigma_{RB}$ is the range bias error, since these errors are the same over a series of track pulses, they will not affect track capability being assessed here.

The range resolution is defined by the pulse width in an unmodulated pulse system and is defined by the bandwidth in a modulated or pulse compression (e.g., LFM) system.

As in the case of range measurements, the measurement accuracy in each angular coordinate is characterized by the rms error, $\sigma_A$, given by the rss of the three error components;

$$\sigma_A = (\sigma_{AN}^2 + \sigma_{AF}^2 + \sigma_{AB}^2)^{1/2}$$

(2)

where the noise angle error, $\sigma_{AN} = \theta/1.6(2(SNR))^{1/2}$, $\theta$ is the radar beamwidth, the factor 1.6 is derived from monopulse angle measurements; $\sigma_{AF}$ is the fixed random error, which will limit angular accuracy for large values of SNR, due to random noise in the receiver angular errors that will be assumed limited to $1/50^{th}$ of the beamwidth; and $\sigma_{AB}$ is the bias error which will not affect short tracks.

Target radial velocity may be measured in one of two ways; either from multiple range measurements or from direct Doppler frequency measurements. The Doppler process will almost always result in better accuracy. The Doppler radial-velocity measurement accuracy is characterized by the rms measurement error, $\sigma_V$, given by the rss of the three error components;

$$\sigma_V = (\sigma_{VN}^2 + \sigma_{VF}^2 + \sigma_{VB}^2)^{1/2}$$

(3)

where the noise velocity error, $\sigma_{VN} = \lambda/2 \tau(2(SNR))^{1/2}$, $\lambda$ is the wavelength and $\tau$ is the duration of the processed waveform; $\sigma_{VF}$ is the fixed error and like the fixed range error case will be assumed limited to the noise error at 20 dB SNR; as with the range and angular errors, the bias error will not be considered for the tracking cases analyzed here.

The radar measurement errors are summarized in Table 2. The LFM range errors were computed for the maximum compressed pulse widths. In the case of the Haystack/HAX radars it is reasonable to assume that, in a debris tracking mode with 10 dB SNR and not a high resolution imaging mode, the limiting range error would be in the order of a meter (as opposed to the 1 to 3 cm error computed, assuming 2 to 1 GHz bandwidths). The velocity errors were computed for a single pulse, assuming Doppler processing. In the debris fixed beam detection mode Haystack has demonstrated the ability to use range-rate measurements to establish estimates of debris inclination. The FPS-85 has the capability to transmit a LFM waveform. As a result it is reasonable to assume the enhanced error accuracy achieved using this waveform would be used in measurements intended to determine a debris orbit.

Table 2. Radar Measurement Errors

<table>
<thead>
<tr>
<th>Noise Error at Max Sensitivity (SNR 10 dB)</th>
<th>Fixed Error at SNR 20 dB &amp; $1/50^{th}$ Beamwidth</th>
</tr>
</thead>
</table>

11
<table>
<thead>
<tr>
<th></th>
<th>Range Error (m)</th>
<th>Velocity Error (m/s)</th>
<th>Angle Error (deg)</th>
<th>Range Error (m)</th>
<th>Velocity Error (m/s)</th>
<th>Angle Error (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FPS-85 LFM (max)</td>
<td>33.0</td>
<td>-</td>
<td>0.18</td>
<td>11.0</td>
<td>-</td>
<td>0.036</td>
</tr>
<tr>
<td>Haystack LFM (max)</td>
<td>0.03</td>
<td>2.0</td>
<td>0.008</td>
<td>0.01</td>
<td>0.65</td>
<td>0.001</td>
</tr>
<tr>
<td>HAX FM (max)</td>
<td>0.02</td>
<td>1.3</td>
<td>0.014</td>
<td>0.005</td>
<td>0.41</td>
<td>0.007</td>
</tr>
<tr>
<td>TIRA (L) Max BW</td>
<td>0.5</td>
<td>-</td>
<td>0.07</td>
<td>0.167</td>
<td>-</td>
<td>0.0014</td>
</tr>
<tr>
<td>Don-2N Max BW</td>
<td>10.0</td>
<td>-</td>
<td>0.037</td>
<td>3.33</td>
<td>-</td>
<td>0.00554</td>
</tr>
</tbody>
</table>

**Orbital Element Errors**

Of the six orbital elements of a piece of orbiting debris, three are most accurately determined by radar measurements. These include the inclination, \( i \), of the debris orbit to the plane of the equator, the longitude of the ascending node, \( \Omega \), and the orbital period, \( T \). The approximate relationships for the 1 sigma errors in these three coordinates, \( \delta_i \) (deg), \( \delta_\Omega \) (deg), \( \delta_T \) (min) have the form\(^\text{23}\)

\[
\delta_i = 0.0123(R \sigma_A \pi/180) + 9.6(R \sigma_R/t_T^2) \quad (4)
\]
\[
\delta_\Omega = 0.0123(R \sigma_A \pi/180) + 9.6(R \sigma_R/t_T^2) \sin i \quad (5)
\]
\[
\delta_T = 0.025(R \sigma_A \pi/180) + 48(R \sigma_R/t_T^2) \quad (6)
\]

where \( R \) (km) is the radar range to the target, \( t_T \) (sec) is the track time, \( \sigma_R \) (km) is the sigma radar range error, and \( \sigma_A \) (deg) is the sigma radar angular error. These estimates seem to be based on empirical data and assume that the target altitude is less than 3000 km, the target eccentricity is less than 0.1 and the radar range is less 2000 km. In the relationships used here it is also assumed that the orbit inclination is greater than 60°, the numerical coefficients have appropriate units to make the equations consistent.

These relationships assume that only range and angle are measured during the track and not range-rate. If range-rate is measured the above relationships become;

\[
\delta_i = \delta_\Omega = 0.0123(R \sigma_A \pi/180) + 1.2(R \sigma_V/t_T) \quad (7)
\]
\[
\delta_T = 0.025(R \sigma_A \pi/180) + 6(R \sigma_V/t_T) + 6(R\dot{\sigma}_V \sigma_V) \quad (8)
\]
where $R_{\text{DOT}}$ (km/sec) is the target range rate, $\sigma_v$ (km/sec) is the sigma radar velocity error.

While these relationships are simple estimates for a rather complex problem, they are adequate to access the basic capability of the current radars to generate track measurements to predict debris orbits. The range, angle only relationships will be used to assess the orbit prediction capability of the TIRA, FPS-85 and Don-2N, since this is the normal operation of the radar. The range-rate relationships will be used to assess the capability of the Haystack and HAX radars, since in the debris collection mode (Pulse CW) the most accurate data would be obtained using range-rate measurements.

**Track Time**

From a review of the orbit prediction relationships it is apparent that radar track time will play a major role in establishing the prediction capability of the radars. The track time available for the TIRA, Haystack and HAX radars in their fixed beam debris collection mode is a function of their beamwidths. For example, a piece of debris orbiting at 500 km in a circular orbit will have an average speed of 0.066 radians/minute. For the Haystack radar with a beamwidth of 0.01° this equates to a track time of 1.59 sec. For an orbit which passes directly through the beam this is 1.59 sec from a point 3 dB below the peak of the beam on one side to a point 3 dB below the peak on the other side. Thus, there will be an effective loss in sensitivity during this equivalent track time. Using the average orbital speeds at other altitudes the TIRA, Haystack and HAX track times were computed. The results are shown in Figure 4. The times shown are independent of the debris size. The minimum detectable size at each track time will be determined by the debris altitude. All larger size debris at this altitude will have the same track time.
The available track time for the FPS-85 and Don-2N array radars is more difficult to determine. Each orbit path through the radar’s surveillance fence will result in a different track length from the point of entry in the fence to the point of exit from the radar’s track field of view (FOV). A continuum of passes for a given orbit could be used to establish minimum and maximum track times available for each orbit. For this assessment it is more important to establish the effect of operating modes (i.e., fence elevation, pulse modulation, etc.) on orbit prediction capability than to assess the optimum performance on any particular orbit. As a result a simple spreadsheet simulation was used to compute the track time available on a single orbit at various altitudes. A 70° circular orbit was selected. The orbit was positioned to enter the surveillance fence on an ascending pass and exit the track FOV such that it passed through the array boresight at 45°. The range to the target at points along the trajectory through the FOV were computed and the equivalent detectable target RCS (and equivalent SEM diameter) determined. The results are shown in Figure 5 for two fence elevations for the FPS-85 simulation. The 25° fence is the normal surveillance fence, which extends from 20° elevation at the edges of the azimuth FOV to 25° elevation at boresight azimuth. The higher fence was positioned to maximize sensitivity, but will provide less coverage. Note that the higher fence does decrease the detectable size at this altitude, but at the expense of track time. The smaller track FOV limits the maximum time to about 70 sec., while the lower fence with less sensitivity at entry doubles the track time.

The results at 1000 km are shown in Figure 6. As was the case at the lower altitude the estimated detectable size has increased for the higher fence location, but at a two to
one decrease in track time. Still a track time over 2 minutes generally results in good prediction accuracies.

![Diagram of track time vs estimated diameter](image)

Figure 6. Available FPS-85 Track Time at 1000 km Altitude

The constant track time for large debris is a result of the single orbit used in this analysis. The total time is from entry in the surveillance fence to exit of the track FOV. This is the constant value shown (e.g., 325 sec in the case of the normal search fence).

Since the Don-2N is nominally a tracking radar, as opposed to a space surveillance radar, a nominal single search fence at 40° was assumed. The results for a lower elevation fence would be the same as illustrated for the FPS-85.

**Orbit Prediction Error**

The basic orbit elements available from the various radar measurements can now be assessed and the errors determined. A recent study evaluated the US radars capability to provide track data sufficient for track to track association. The major component of error in all these evaluations was the period error. As a result only the orbital period error will be evaluated here.

**Period Error**

Using the relationship given in (6) the period error was computed for the FPS-85 and the Don-2N phased array radars for a 70° circular orbit at 500 km. For a 40° elevation debris surveillance fence, the results are shown in Figure 7.
Due to the greater sensitivity of the Don-2N at lower altitudes (see Fig. 3) it achieves a 0.1 min period error on debris roughly half of size of that achievable with the FPS-85.

The period error was also computed for both radars at a 1000km altitude. These results are shown in Figure 8. Due to the higher sensitivity of the FPS-85 at the higher altitudes (see Fig. 3), the capability of the radars in terms of period error versus debris size reversed. The FPS-85 now provides the lower error for a given debris size. Note that these are single pulse results. It would be expected that at the higher altitudes both radars would attempt to provide pulse integration during track.
Using the relationship given in (6) the period error was computed for the TIRA, Haystack and HAX radars.

Haystack and HAX have the capability to transmit and process LFM pulse compression waveforms. While not the most effective waveform for fixed beam debris measurements, it is the more effective waveform for tracking. Also the full bandwidth capability of the radars, while necessary for imaging, is not required to achieve accurate range measurements. As an example, a 7 MHz LFM waveform would produce a sigma range error of about 5 m. Considering a 1 millisecond transmit waveform this represents about a 7000:1 compression ratio. These are reasonable parameters to suggest using in a Haystack and HAX debris tracking mode.

It is unlikely that Haystack would ever attempt to operate in a stare and chase surveillance mode given the size of the antenna and its slew rate capability. It could, however, operate in a cued search mode, given a crude element set. In this mode, the LFM track mode would be beneficial as well. The LIDAR and HAX are much more likely to be able to operate in a stare and chase mode. In this mode it is reasonable to assume that a minimum 30 sec. track time could be obtained. With these assumptions the period error for the three dish radars is shown in Figure 9.
At the lower altitudes all three radars are capable of providing excellent prediction for debris in the 1 to 2 cm class. The results at a higher altitude are shown in Figure 10. Here the effect of the lower sensitivity of the HAX is evident. For the HAX some pulse integration on track will improve the detectable debris size. Since the TIRA is operating in the Rayleigh below an estimated debris size of 4 cm, integration will not improve the sensitivity performance.
Small Debris Cataloging

To be able to avoid collision with 1 to 10 cm debris the element sets of the debris must be in the US Space Surveillance catalog. The catalog is maintained by a process of tasking radar to provide tracking data, processing the data, updating the debris element set and repeating the process. All tracks (observations) are correlated to the catalog. They either match and the update follows or they do not and are declared an Uncorrelated Target (UCT). Correlation with previously tracked UCTs is then made, and the element set is either updated and the correlation process with the catalog continued, or they are entered as another UCT\textsuperscript{25}.

The biggest challenge associated with tracks on small debris, is reacquisition on subsequent passes. Reacquisition must be attempted at the first available opportunity, specifically the first pass following the initial detection. Finding small debris after several orbits can be difficult due to atmospheric drag. Initial UCT element set accuracy should be accurate enough that other sensors in the network can acquire and track the debris. If other tracks are made, the debris object may correlate with other UCT tracks of the debris and enter the catalog.

The criteria to determine track status is associated with the comparison of the estimated position of the debris object with those in the catalog. Correlation occurs if the object is within the association volume. The association volume used by the US Space Command for associating tracks of known objects is a three dimensional box in the in-track, radial and out-of-plane position space centered on the predicted position. The nominal sides of this box are\textsuperscript{26}:

\begin{itemize}
  \item In-track: 3 seconds (0.05 min)
  \item Radial: 5 km
  \item Out-of-Plane: 0.05 deg
\end{itemize}

If outside this volume, the new UCTs (new debris tracks) will then be compared with other UCTs to determine if any UCTs correlate, as noted. The correlated UCTs can then be combined to develop a catalog entry sufficient for tasking updates from additional radars (if possible). Currently the criteria for UCT correlation can be 3 to 4 times that for catalog correlation (e.g., 0.2 min in-track, 0.2 deg. inclination). As seen in Figures 7 through 10, the radars being evaluated here can provide UCT association at altitudes below 1000 km in their various cued and un-cued track modes. They cannot provide highly accurate catalog correlations in most cases.

It is envisioned that, based on the increased density of space objects in the region from 1 to 10 cm, the number of tracks (observations) and the processing required will increase dramatically. The criteria for a new uncorrelated target entry must be met to insure that false observations are not placed in the UCT file. Current criteria of the orbit parameters listed above were established primarily on the basis of detecting and tracking 10 cm to 1 meter objects (payloads and rocket bodies and large debris) and the nearest neighbor distance between these objects. If cataloging of debris objects in the 1 to 10 cm
range is required, it might be necessary to establish unique criteria for this range. Criteria must be defined based on the density of the 1 to 10 cm population at the altitudes of interest.

**LEO Debris Environment – ORDEM2000**

The NASA ORDEM 2000 Model was used\(^1\) in an effort to characterize the LEO debris environment. This model used the Haystack debris campaign of 2003 and other radar data to build a LEO debris environment model to describe the debris spatial density and velocity distribution in space. The ORDEM2000 debris environment model describes the spatial density, velocity distribution, and inclination distribution of debris particles at different latitudes and altitudes. The debris environment is represented by a set of preprocessed data files. No assumptions regarding debris particles’ inclinations, eccentricities, or orientations in space (longitudes of the ascending node and arguments of perigee) are required in this approach. However, a decision was made to randomize the objects’ ascending node longitudes. For an observer using radar to observe orbital debris from the ground, two options are available: a vertical staring mode and an arbitrary pointing mode.

As an example, FPS-85 was used in a test case using the arbitrary pointing mode. The user also has the option to obtain the surface area flux of any arbitrary size objects. The surface area flux is defined as the number of objects passing through the beam width, per unit area, per unit time. The inclination distribution and velocity distribution files at various altitudes are also available for additional analyses. The arbitrary pointing mode requires four parameters: the geographic latitude of the instrument, the time of the observation (between 1991 and 2030), and the pointing direction in terms of azimuth and elevation angles. The standard output is the surface area flux of particles of the six standard fixed sizes (>10 µm, >100 µm, >1 mm, >1 cm, >10 cm, >1 m). The user also has the option to obtain the surface area flux of any arbitrary size objects. This data in can be used to determine the number of objects in a search sector of the radar. The number in a single search cell will be the product of the flux values, the search pulse range cell, and the surface area of the frustum of the conic search beam.

For the FPS-85 at 30 deg Latitude, 45 deg elevation, and 180 deg azimuth the number of objects in a search beam was computed at four altitudes. The results are shown in Figure 12 for the FPS-85 0.25 millisecond pulse and 1.3 deg beamwidth. From 10 cm to 1 cm the number of objects increases about an order of magnitude. From 10 cm to 5 cm the increase is in the order of 2. There is debate as to over what size range the current UCT association criteria applies. If the current criteria based on nearest neighbor distance applies to only 10 cm objects and above, there could be problem in using it below 5 cm. Since the FPS-85 without integration should be able to collect track data on objects in the 4 cm to 6 cm range with good orbit prediction accuracy, the radar data could provide a good test data for criteria selection.

---

\(^1\) A download of the model is available at; http://orbitaldebris.jsc.nasa.gov/model/engrmodel
Figure 12. Objects in a single FPS-85 search cell per orbit period

Estimated Debris Separation

In an attempt to estimate the nearest neighbor distances represented by the number passing through a cell, the standard Poisson Probability Distribution problem for traffic flow was applied to the flux data. This seems reasonable since the ascending nodes are randomized. Figure 13 gives the results for an altitude of 1000 km. The distribution for the > 1 cm region was computed using a rate of 36.336 objects per orbit period (105.1 min). The distribution for the > 10 cm region was computed using a rate of 3.912 objects over the same orbit period. At this altitude the single orbit period is adequate to show the distribution about the mean values of 425 km in the > 10 cm region and about 35 km in the > 1 cm region. The separation in mean values (and distributions) shows the impact of the larger difference in debris densities at this altitude. Even in the > 10 cm region there is a reasonable probability of seeing object separations in the order 200 km.
Consistent with the flux data findings there is roughly an order of magnitude difference in the estimated separation distances between the two regions. Of major concern, if these estimates are correct, is the small separations estimated for the 1 cm region.

Note that these separation estimates are made without the application of velocity and inclination filtering. Based on the close proximities derived, a more expansive investigation of the ORDEM2000 data base is required to account for the velocity and inclination characteristics of the debris environment.

The impact of this flux analysis on the association can only be estimated. Since actual Haystack measured data and other radar data were used in assembling the environmental data base, it is hoped that actual radar track data on orbit elements of the small debris population can be used in assessing the association criteria for this region. Based on the data available, the FPS-85 and Don-2N presents the most likely sources to acquire and track debris in the region of interest. Acquiring and analyzing track data in the 5 cm to 10 cm range would be a first step in assessing the association criteria for small debris. Both radars should be able to acquire such data without modification in the 500 km to 600 km altitude region. Finally, the practice of expanding the association criteria, particularly for small object UCT, to UCT correlation might not be the best approach. Use of the FPS-85 and Don-2N in a debris track collection mode would also help answer the question of the effects of traffic density on available radar search and track times, pulse selection and integration times.

Figure 13. Estimated Debris Search Cell Separation Distance – 1000 km
Summary of Current Capability

Based on the brief analysis conducted here and the parameters estimated for the FPS-85 and Don-2N phased arrays, these radars have the capability to construct a high elevation “debris fence” to provide un-tasked detection and tracking of small debris. With the high fence the radar is able to provide measurements accurate enough to meet current UCT correlation criteria on 4 cm debris and larger to 1000 km. This conclusion is based on using a single pulse. The sensitivity can be improved by a small degree by pulse integration to allow smaller debris to be tracked, but at the expense of multiple target detection and tracking. This option was not evaluated.

The Haystack, HAX and LIDAR radars offer the required frequency of operation and sensitivity to detect small debris. Their basic limitation for track is their narrow beamwidths and their fixed beam operation, which significantly limits track time. They do not offer a surveillance-track capability. However, they do provide a means to validate small debris orbit predictions in a cued search mode. It has been shown\(^2\) that HAX, depending on antenna slew rates, has the sensitivity to provide detection and tracking of small space debris in a cued stare and chase mode. Since it was designed as a debris detection and track radar, it is much more likely to be available for debris measurements programs than the highly utilized Haystack. With on-pulse modulation (LFM) and 30 sec track times HAX can provide accurate range and angle measurements to meet UCT correlation criteria, as presently defined, for 2 cm and larger debris out to 1000 km. With cueing the Haystack radar with LFM on-pulse modulation and at least 30 sec track times can update all small (1 to 10 cm) debris UCT element sets, with period and inclination prediction accuracy to meet all current UCT correlation criteria over the altitude range of interest. LIDAR has demonstrated debris measurement capability comparable to HAX. With cueing the radar and 30 sec tracks the radar can meet UCT correlation criteria on 2 to 4 cm objects over the altitude range of interest.

Observations/Recommendations

Future debris surveillance radar designed to catalog 1 to 10 cm debris for collision avoidance should operate in the S-band to C-band region and have the sensitivity to detect/track 1 cm targets at 1800 km. Ideally the radar should have agile beam capability to simultaneously search and track multiple debris targets to 1800 km with a minimum track time of 60 sec.

A small debris catalog criteria must be established to correlate new debris UCTs for the high density (1 to 10 cm) altitude regimes. As a first step in the development of such criteria simulations, small debris data collection campaigns should be initiated using existing radar assets. A key element in these campaigns should be the data exchange between the widely dispersed radars identified in this report. As additional debris radar test sites come on line, the sharing of their data could be a major contributor to this effort.
The Proposed US Air Force S-band Space Fence Concept might meet the debris tracking radar requirements and form the main element of a Space Debris Surveillance Network. The FPS-85 and Don-2N array radars have the potential to contribute to a Space Debris Surveillance Network as secondary sensors. Haystack, HAX and LIDAR can provide RCS measurements and updates on established element set data when tasked. If proven capable, the ESA test radars could also be a major contributor to the Network.

References

6. D. Messier, “ESA Developing Space Safety Radar to Track Orbit Debris”, parabolicare.com/2012/09/14/esa

*All reference material is available on-line at Google Search, except the Workshop Records*
22. Radar System Performance Modeling – Powered by Google, Chapter 8, Radar Measurement and Tracking