Improving LEO prediction precision with TLEs

LIU Wei, WANG Ronglan, YAN Ruidong, GONG Jiancun

(1. Center for Space Science and Applied Research, CAS, Beijing 100190, China)

Abstract: TLE is the only publicly available and most complete data sets of Earth orbit objects. The prediction accuracy with SGP4/SDP4 is limited. The TLE+SGP4/SDP4 prediction error sources and the theory of improving it with historical TLE data were analyzed. A pseudo-observation data generation method was introduced briefly. The span of TLE data used is dependent on the orbital altitude. Two objects were selected and the orbit was predicted with an improved TLE+numerical method. The statistical results show that the improved TLE forecasting accuracy is greatly improved. Finally, the application of the improved TLE and some possible problems were discussed.

Key words: TLE data; accuracy improvement; fitting period

0 Introduction

Two-Line Element Sets (TLE) are published by NORAD for all resident space objects. These element sets are periodically refined and the only publicly available and the most complete cataloging Earth orbit space object cataloging data sets. TLEs are “mean” value obtained by removing periodic variations in a particular way. In order to obtain good predictions, these periodic variations are reconstructed by the SGP4/SDP4 model[1]. However, TLE+SGP4/SDP4 prediction accuracy is limited.

The SGP4/SDP4 model was based on the Brouwer theory, considering J_1 gravitational model and a exponential atmospheric density model and the Moon and Sun gravitational effects[2]. The drag effect was obtained from SGP4/SDP4 drag term B_. Some work had been done about TLE+SGP4/SDP4 prediction errors[3-6]. TLE+SGP4/SDP4 error source was composed of two parts: TLE initial error and SGP4/SDP4 model error. As for how to improve the forecast accuracy using TLE, domestic and foreign scholars have done some research. Literature[7, 8] validated the improved TLE prediction results with four laser ranging satellites, whose orbital altitudes were above 800 km. In this paper, the forecast error composition, properties and the improvement theory were discussed. Using historical TLE pseudo-observation data, we got the relation between the orbital altitude and the fitting period and assess the fitting result. The results show that the fitted TLE forecast accuracy was improved to a great extent, especially in the case of a prediction period more than one day. It is important to improve the collision warning confidence. Finally, the application of improved TLE and some possible problems were discussed.

1 Improvement principles and process

TLE improvement is based on the SGP4/SDP4 model, which considered only fourth-order flat rate perturbation and exponential atmosphere model, ignoring the higher-order periodic perturbation terms. This caused the prediction error performed as variances. This section gave the improvement principles and process.

1.1 TLE+SGP4/SDP4 error source analysis

The TLE+SGP4/SDP4 main error source was composed of two parts: One is TLE initial error performed as a bias in Fig.1, caused by the measurement errors and the model errors in the orbit determination. The bias error cannot be removed without more accurate measurement data. The other part is the SGP4/SDP4 model error
performed as variances in Fig.1. The variance is essentially a random error and can be removed by the statistical method of least squares or Kalman filtering in the orbit fitting process. In addition, the exponential atmospheric density model errors in SGP4/SDP4, cause greater forecast error during the solar maximum period. However, the fitted TLE+high-precision numerical propagator can reduce atmospheric density errors in the orbit propagation.

1.2 Improvement process

Based on the previous analysis, pseudo-observational data were generated by forward and backward forecast historical TLE data that combine station coordinates and target visibility criterion. Assumed that $\sigma_{t_0}$ was the latest orbit element at epoch $t_0$ and $\sigma_{t_0}, \sigma_{t_1}, \ldots, \sigma_{t_n}$ were historical TLE data in Fig.2.

The measurement equation was listed as follows:

$$\rho_{po} = G(t, X) + V$$

The measurement equation has Taylor expansion at the reference value $X_0^*$, omitting higher order terms then the differential correction equations were:

$$y = Gx + v$$

where: $y = \rho_{po} - \rho_e$ is residual, $x = X_0 - X^* = X_0^k - X_0^{k-1}$ is correction value.

We got the final status $X_0^k = X_0^* + x$ by continuous iteration. The theory of precision orbit determination was listed in literature \cite{9}.
The Main models considered in the fitting process:
(1) 70×70 degree gravitation model GGM02C;
(2) 30×30 degree TOPEX4.0 ocean tide model;
(3) NRLMSIS-00 atmosphere model;
Solar radiation, solid earth tide, pole tide and N body perturbation were also considered in the fitting process. The coordinate translation and the time system used the IERS Conventions (2003) standard. Finally, the osculating elements $\bar{\sigma}$ at epoch $t_0$ were obtained, then predicting the orbit with the high-precision numerical propagator. This is the whole process of TLE improvement.

2 The Fitting period determination

In the process of generating pseudo-observational data, the historic TLE data $\sigma_{n}$ were used. The value of $t_n$ will affect the fitted result. We call $t_0 - t_n$ the fitting period in Fig.3.

The orbit fitting residual consists of two parts. The initial orbital elements error accumulate and diverge in the numerical orbit propagation. The main model error type and magnitude depend on the orbital altitude. The fitting period is not the longer the better, when the fitting period exceeds a certain threshold, the corrected value of the orbital elements will absorb large amounts of the model error. Therefore, the determination of the fitting period is the result of a compromise of the initial elements error and the model error. The atmospheric model error is the main error source for objects with an altitude below 1000 km, and it is larger than other model errors. The atmospheric density exponentially decreases sharply with the orbital altitude increasing, the corresponding error is smaller, so the fitting period can grow some.

In order to quantitatively get the relationship of the fitting period and orbital altitude, two satellites were tested in table 1. The first satellite orbital altitude is about 350 km, and the other is ajsiai, whose orbital altitude is about
1490 km.

Table 1. The main parameters of selected satellites

<table>
<thead>
<tr>
<th>NORAD</th>
<th>Altitude</th>
<th>Eccentricity</th>
<th>A/M</th>
</tr>
</thead>
<tbody>
<tr>
<td>37820</td>
<td>350km</td>
<td>&lt;0.0020</td>
<td>0.00414</td>
</tr>
<tr>
<td>16908</td>
<td>1490km</td>
<td>&lt;0.0012</td>
<td>0.0053</td>
</tr>
</tbody>
</table>

We took the reference orbit as "truth" to get the forecast error. The GPS ephemeris was the reference orbit of object 37820. The improved results were similar when the fitting period selected was 1 or 2 days. However, the result was worse when the fitting period changed to 3 days. It also verified the conclusion that when the fitting period exceeds a certain threshold, the correction value of the orbital elements will absorb a large amount of model error. For a near-Earth orbit, 1 or 2 day fitting period is appropriate.

![Fig. 5 16908 Prediction error with different fitting period](image)

The result of precise orbit determination with laser ranging data, whose residual RMS is about a centimeter as with the object 16908 reference orbit. The forecast accuracy was best among three situations, when fitting period selected 10 days. With the fitting period increasing, the forecast accuracy became better in the fitting process. For object 16908, a 10 day fitting period is appropriate.

3 Improvement accuracy assessment

In order to further assess the forecast accuracy of improved TLE, two satellites above were tested. UNW (along-track U, cross-track, and N, normal to the orbital plane W) errors were given in five different independent time spans by comparing the fitted TLE+numerical method results with precise ephemeris and comparing the TLE+SGP4 results with the precise ephemeris.
Fig.6  37820 U forecast error statistics

Fig.7  37820 N forecast error statistics

Fig.8  37820 W forecast error statistics
From Fig. 6-9, it is obvious that the forecast results of fitting the TLE was significantly better than that of TLE+SGP4/SDP4. For TLE+SGP prediction for 6 days, the maximum error in along track was up to 13 km, the average value was 6.5 km. In the fitting case, the maximum error in along track was up to 2.6 km, the average value was 1.3 km.

The determination of the error ellipsoid in the collision probability calculation becomes difficult, reducing the early warning confidence. The forecast accuracy has marked improvement and smaller fluctuations in the fitting case. It is very advantageous to improve the early warning confidence.

TLE+SGP4/SDP4 model prediction for 30 days had a better error control in the UNW direction. For the SGP4/SDP4 models only take into account the fourth-order flat rate perturbation, ignoring the higher order periodical impact. The forecast error was very unstable, so the error had very large fluctuations. This can be seen in the box plot Fig.10-13.
Fig. 11 16908 N forecast error statistics

Fig. 12 16908 W forecast error statistics

Fig. 13 16908 Position forecast error statistics
4 Summary

In this paper, the forecast error composition, properties and improvement theory were discussed. Using historical TLE pseudo-observational data, the relation between the orbital altitude and the fitting period were obtained and assessing the fitting result. The results show that the fitted TLE forecast accuracy was improved greatly and the prediction errors were stable, especially in the case with a prediction period more than one day. It is important to improve the collision warning confidence. Finally, we discussed the application of the improved TLE and some possible problems.

One problem is that the determination of the non-cooperative object’s area-to-mass ratio (A/M). Its initial values could be obtained from B* or the dynamic evolution of the semi-major axis. Then take it as a parameter to be corrected in the fitting process. However, the initial elements error and A/M error will absorb with each other. The subsequent orbit prediction will, therefore, introduce new errors, and also limits the length of the forecast time span. In addition, removing the abnormal TLE data is also to be considered.

In the future, we will select more space objects for test and accumulate more experience for its application. To search for an effective method for determining the area-to-mass ratio of non-cooperative target is also very important.

References: