Uncertainty in Probability of Collision Calculations

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12 April 2013
Collision Prediction & Covariance

- Collision prediction is dependent upon three covariance based processes:
  1. Covariance generation
  2. Covariance propagation
  3. Covariance assessment, probability determination and miss distance calculations

- Numerous techniques have been developed to address the propagation and assessment processes, yet detailing the components of the covariance generation process has been neglected despite dominating the accuracy and confidence of any resulting collision calculation
  - Lack of or poor or wildly varying covariance data results in extremely conservative/pathological predictions impacting planning, operations and resources
  - Optimistic covariance data significantly increases the risk of a collision

- By understanding the sources, dependencies, sensitivities and overall statistical structure of an object’s covariance additional information can be supplied to accurately assess the probability of collision between two objects
  - Sensor & measurement characteristics
  - Modeling uncertainties
  - Systematic errors & biases
Covariance Components: Optimistic & Realistic

- As a statistical measure covariance is directly affected by and reflects the metrics of the observations used to generate the solution
  - Observation type, quantity and quality
  - Geometry of collection

- Solution or solved for only based covariance provides an optimistic or best case value for the uncertainty since it is comprised solely of measurement noise
  - Implemented in most simulations and processes

- To account for systematic errors, biases, model uncertainties and overall system complexity, a second covariance matrix including these additional terms to consider is added to the solved for only solution
  - Many simulations and processes ignore this additional noise or have it set incorrectly

- The combined solved for and consider covariance matrices represent a more realistic representation of object’s uncertainty
  - Resulting covariance can have dramatically different orientation and magnitude
  - The values for consider terms are determined from calibration
Covariance Considerations

- Increasing the quality, quantity and information (e.g. include ranging data with angle measurements or better geometry) improves the resulting measurement based covariance solution directly
  - Increased measurements reduces the noise in the solution by \((n)^{\frac{1}{2}}\)
  - 10x improvement in ranging quality results in ~10x overall improvement in the covariance
  - Favorable viewing geometry

- Similarly, the inclusion of error parameters and system noise characteristics results in up to an order of magnitude increase in the combined covariance over the measurement based covariance due to:
  - Sensor location errors
  - Sensor biases & corrections
  - Timing errors
  - Modeling errors and uncertainties
  - System complexity
Covariance Analysis Test Case

- The failed launch test case was developed, to illustrate the dependencies and sensitivities inherent in generating a combined covariance matrix against a challenging LEO target
  - Single LEO object with no a priori trajectory data
    - Vary altitude from 400 to 1100km
  - Initial over flight of a single ground site (radar or telescope)
  - Determine the solved for and combined covariance as a function of elevation and number of observation by varying and including the following:
    - Observation metrics:
      - Type: Active (radar: azimuth, elevation and range) vs. Passive (telescope: angles only)
      - Rate: 1 point/s, 10 points/minute, 5 points/minute
      - Quality (1 sigma):
        - Active: 0.01 degrees for angles and 10m for range
        - Passive: 10 arc-seconds
      - Geometry: Vary over flight duration from horizon maximum elevations spanning 5-85 degrees
    - Consider Parameters:
      - 1 m site uncertainty
      - 1 micro second timing error
      - model uncertainties as reported by EGM-96
      - Sensor bias uncertainties at 10% 1 sigma values
      - 3% drag uncertainty per NRL MSIS-00
  - Analysis Products
    - RSS Solved for Only and Combined Covariance output 1 rev predict
    - Duration of overflight varied by ob rate from rise until set
  
  SIMULATION PROCESS:
  - GENERATE OBSERVATIONS WITH NOISE CHARACTERISTICS
  - GENERATE COVARIANCE
  - PROPAGATE COVARIANCE

  SIMULATION SETUP:
  - SOLVE FOR CARTESIAN STATE AT RISE
  - 41X41 GEOPOTENTIAL PER EGM-96
  - NO SOLID EARTH TIDES
  - SOLAR AND LUNAR PERTURBATIONS
  - DRAG PER NRL MSIS-00
  - BALLISTIC COEFFICIENT 0.015 M^2/KG

  VARY:
  - ALTITUDE
  - OBSERVATION RATE
  - TOTAL NUMBER OF OBS IN SOLUTION
  - SOLVE FOR DRAG VS. CONSIDER DRAG
Distribution of LEO Satellites
LEO Over Flight Duration – Rise to Set

Graph showing the relationship between time above 3 degree elevation and maximum elevation. The graph compares two trajectories: 400 km Circular and 1100 km Circular.
## Summary of Results: Active Tracking

### END OF PASS RSS 1 SIGMA COVARIANCE (KM)

<table>
<thead>
<tr>
<th>Altitude (KM)</th>
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<th>25</th>
<th>35</th>
<th>45</th>
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### 1 REV PREDICT RSS 1 SIGMA COVARIANCE (KM)

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<th>Altitude (KM)</th>
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**Legend:**
- <50 M
- >500 M
- >5 KM

Consider terms increase covariance from 2x to greater than 10x measurement only based solution.
Summary of Results: Passive Tracking

### END OF PASS RSS 1 SIGMA COVARIANCE (KM)

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<th>ALTITUDE (KM)</th>
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### 1 REV PREDICT RSS 1 SIGMA COVARIANCE (KM)

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<td>MAX ELEVATION (DEG)</td>
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**Legend**

- <50 M
- >500 M
- >5 KM

**Consider Terms Increase Covariance Up to 2x Over Measurement Only Covariance**

INCLUDE BIASES AND MODELING ERRORS
Covariance: 400 km Active vs. Passive Tracking

![Graph showing covariance comparison between active and passive tracking methods. The graph illustrates the uncertainty in position over time for different tracking scenarios. The x-axis represents time from rise in seconds, and the y-axis represents position uncertainty (1 sigma) in kilometers. The graph includes lines for passive end of pass, active end of pass, and active 1 rev predict.]
Covariance Comparison: 400km Optimistic to Realistic

45 DEG MAX ELEVATION

- **MEASUREMENT ONLY**
- **CONSIDER PARAMETERS**
- **DRAG + CONSIDER PARAMETERS**

**Legend:**
- **OPTIMISTIC REV 1 PREDICT**
- **REALISTIC REV 1 PREDICT**
- **REALISTIC + DRAG REV 1 PREDICT**
Collision Geometry & Probability Calculations

- K. Chan details the calculating the actual intersection volumes
  - Modeling the actual collision is challenging and numerically intensive
  - For high speed collisions the geometry can be simplified to include only the cross sectional area orthogonal to the velocity vector
- A further simplification can be made per Alfriend to compute the resulting probability of collision as a function of the collision angle and the area

<table>
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<th>(METERS)</th>
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<td>DRAG + CONSIDER</td>
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<td>588.83</td>
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TRUE COLLISION GEOMETRY

SIMPLIFIED COLLISION GEOMETRY

COVARIANCE CROSS-SECTIONAL AREAS TO SCALE
Covariance Components & Probability of Collision

Spacecraft Position Knowledge: 1 m
Spacecraft Size: 5m
Assessed Miss Distance: 150m
Encounter Angle: 0 – 180 deg

Failed Launch Covariance:

Operational Spacecraft

Covariance Ellipsoids to Scale

Failed Launch

Measurement Only

1/2 of Consider Only Volume

1/10 of Drag + Consider Volume
Summary & Future Work

- Realistic covariance generation requires the inclusion of biases, errors and uncertainties

- System and sensor calibration is required to generate inputs for accurate covariance generation

- Use of or sharing incomplete, inconsistent or optimistic covariance results in increasing the uncertainty of whether a collision will occur

- Additional test cases to be evaluated:
  - Failed GTO
  - Re-entry
  - Anomalous maneuver (eccentricity)